MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2017/2018 ACADEMIC YEAR
THIRD YEAR *FIRST SEMESTER*

SCHOOL OF SCIENCE &INFORMATION SCIENCES
BACHELOR OF SCIENCE-MATHEMATICS

COURSE CODE: MAT 320

COURSE TITLE: DYNAMICS

DATE: 16th APRIL 20 18 **TIME: 11-1pm**

INSTRUCTIONS TO CANDIDATES

1. Answer Question **ONE** and any other **TWO** questions

This paper consists of ***3*** *printed pages. Please turn over.*

SECTION A (COMPULSORY)

Question One (30Mks)

a)A thin uniform rod has length l and total mass M and the linear mass density varies with the distance x from the left according to $\lambda = \frac{\lambda_o}{l}x$ where λ_o is a constant with units in Kgm^{-1}. Determine an expression of λ_o in terms of l and M. Find the center of mass. **5mks**

b) Let there be a general direction $\overrightarrow{OM}$ around which a vector $\overrightarrow{OA}$ of constant magnitude rotates with a constant angular velocity $\underset{\sim}{\omega}$ in a fixed frame. Show that $\frac{d\underset{\sim}{A}}{dt} = \underset{\sim}{\omega} \times \underset{\sim}{A}$. **5mks**

c) The mean distance from the earth to the sun is $r_{e,s} = 1.49 \times 10^{11}$ m. The mass of the earth is $m_e = 5.95 \times 10^{24}$ kg and the mass of the sun is $m_s = 1.99 \times 10^{30}$ kg. The mean radius of the earth is $r_e = 6.37 \times 10^{6}$ m. The mean radius of the sun is $r_s = 6.96 \times 10^{8}$ m. Where is the location of the center of mass of the earth-sun's system? Explain your answer. **5mks**

d). A particle moving with an initial velocity of $50ms^{-1}$**j** undergoes an acceleration **a**$=[35ms^{-2}+(2ms^{-5})t^3]$**i**$+[4ms^{-2}-(1ms^{-4})t^2]$**j.** What is the particle's velocity after 3.0 seconds assuming that it started from the origin? **4mks**

e) Given a pendulum made of a spring with a mass m on the end with a spring arranged to lie in a straight line with equilibrium length ℓ and let the spring have a length $\ell + x(t)$ with a vertical angle $\theta(t)$. Assuming that the motion takes place in a vertical plane, Find the equations of motion for x and θ.

5mks

f) A particle of mass **2** units is at position $\mathbf{r} = t^2\mathbf{i} + t^3\mathbf{j}$ relative to a fixed frame S. If the origin of S' is moving along the vector $\mathbf{R}(t) = (t^2+3)\mathbf{i} + (t-3)\mathbf{j}$ relative to S, calculate:

i) $\mathbf{v}'$ **4mks**

ii) $\mathbf{a}'$ **2mks**

SECTION B: ANSWER ANY TWO QUESTIONS

Question Two (20Mks)

a) Using the case of one observer in an inertial frame and another in a moving reference frame, derive the Lorentz Transformations $x' = \dfrac{x-ut}{\sqrt{1-\dfrac{u^2}{c^2}}}$ and $t' = \dfrac{t-\dfrac{ux}{c^2}}{\sqrt{1-\dfrac{u^2}{c^2}}}$ given that the observers assign an event the coordinates (x,t) and (x',t'), respectively. **(10 Marks)**

b) Consider a function $x(t)$ for $t_1 \le t \le t_2$ which has its end points fixed, and a quantity, $s = \int_{t_1}^{t_2} L(x,\dot{x},t)dt.$ If the function $x_o(t)$ yields a stationary value (that is, a local minimum, maximum or saddle point) of s, show that $\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_o}\right) = \dfrac{\partial L}{\partial x_o}$.

10mks

Question Three (20Mks)

Consider two frames of reference S and S' with unit vectors $n = (\mathbf{i},\mathbf{j},\mathbf{k})$ and $n' = (\mathbf{i}',\mathbf{j}',\mathbf{k}')$ and with a common origin. Let S' rotate with some axis through the origin with angular velocity ω. Given a particle p whose position vectors are $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $\mathbf{r} = x'\mathbf{i}' + y'\mathbf{j}' + z'\mathbf{k}'$ relative to the frames S and S' respectively?

i) Show that $\mathbf{v} = \mathbf{v}' + \underset{\sim}{\omega} \times \mathbf{r}$, where $\mathbf{v}$ and $\mathbf{v}'$ are expressions of velocity vectors in frames S and S' respectively. **10mks**

ii) Obtain the expression for acceleration in both frames and by use of Newton's Second Law of motion; obtain the expressions for Coriolis and Centrifugal forces. **10mks**

<u>Question Four (20Mks)</u>

Two particles A and B of masses 2kg and 3kg respectively are at position vectors $\mathbf{r}_A = (2t^2 + t + 1)\mathbf{i} + (3t + 4)\mathbf{j} - 8\mathbf{k}$ and $\mathbf{r}_B = (4t^2 + 4t)\mathbf{i} + (t^4 + 3t)\mathbf{j} + (3t - 4t^2)\mathbf{k}$.

Calculate at $t = 1$ s

i) The Centre of mass of the system. **4mks**

ii) Total momentum of the system. **4mks**

iii) Angular momentum of the system. **6mks**

iv) Kinetic energy of the system. **6mks**