MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS

2023/ 2024 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES.

MASTERS (APPLIED STATISTICS)

COURSE CODE: STA 8215

COURSE TITLE: TIME SERIES ANALYSIS

DATE: **TIME:**

<u>INSTRUCTIONS TO CANDIDATES</u>

Answer Question ONE and any other TWO questions

This paper consists of THREE printed pages. Please turn over.

QUESTION ONE (20 MARKS)

a) Define the following:

i. White noise (2 mks)

ii. A process Y_t is strictly stationary. (2 mks)

iii. A process Y_t is 2nd order stationary or weakly stationary. (2 mks)

b) Given that a time series time series process is given by :

$$X_t = U_1 Sin(2\pi\lambda_0 t) + U_2 Cos(2\pi\lambda_0 t)$$

Where U_1 and U_2 are independent, mean zero and variance σ^2 random variables. λ_0 is the frequency of the process. Show that the autocorrelation γ_h is given by:

$$\gamma_h = \frac{\sigma^2}{\alpha}(e^{-2\pi i\lambda_0 h} + e^{2\pi i\lambda_0 h})$$ (10 mks)

c) In Box-Jenkins approach for fitting an ARIMA model one part which is important in identification? Explain and describe the process. (4 mks)

QUESTION TWO (20 MARKS)

(a) Given the AR (1) process:

$X_t = \alpha X_{t-1} + e_t$, given $e_t \sim N(0, \sigma^2)$. Show that:

i. $Var\ (X_0) = r_0 = \frac{\sigma^2}{1-\alpha^2}$ (3 mks)

ii. $r_k = \alpha^{[k]} r_0 \quad for\ k \neq 0$ (5 mks)

iii. From the results in a (i) and a (ii) that $r_0 = \frac{\sigma^2}{1-\alpha^2}$ and $r_k = \alpha^{[k]} r_0 \quad for\ k \neq 0$, show that the spectral density function distribution function (spectrum), $f(\lambda)$ is given by:

$$f(\lambda) = \frac{\sigma^2}{2\pi(1 - 2\alpha\ Cos\ \lambda + \alpha^2)}$$

Where $e^{-i\lambda} + e^{i\lambda} = 2Cos\ \lambda$ (9 mks)

(b) Given that an AR(1) is $y_t = \rho y_{t-1} + \varepsilon_t$. Using repetitive definition of AR(1) show that $y_t = \rho y_{t-1} + \varepsilon_t = \sum_{j=0}^{\infty} \rho^j \varepsilon_{t-j} + \rho^k \varepsilon_{t-k}$ (3 mks)

QUESTION THREE (20 MARKS)

(a) List applications of time series. (6 mks)

(b) Given a time series process X_t such that $X_t = A\ Cos\ (\theta t) + B\ Sin\ (\theta t)$ where A and B are uncorrelated variables with zero mean and unit variances $\theta \in (-\pi, \pi)$ and $\mu_x(t) = 0$. Show that $\{X_t\}$ is a stationary process. (8 mks)

(c) Define an ARMA (p, q) process $\{X_t\}$. (2 mks)

(d) Explain the following:

i. A process $\{X_t\}$ is said to be causal. (2 mks)

ii. A process $\{X_t\}$ is said to be invertible. (2 mks)

QUESTION FOUR (20 MARKS)

(a) Prove that $r_k(h) = E\left[\left(\sum_{j=-\infty}^{\infty} \varphi_j W_{t+h-j}\right)\left(\sum_{j=-\infty}^{\infty} \varphi_j W_{t-j}\right)\right] = \sigma^2 \sum_{j=-\infty}^{\infty} \varphi_{j+h} W_j$

Where $W_t = \varepsilon_t$, is the error term $(i.e. W_t \sim N(0, \sigma^2)$. (7 mks)

(b) Describe the properties of $\bar{X}_n, \hat{r}_k\ and\ \breve{\rho}_x(h)$ (13 mks)