MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATION
2023/2024 ACADEMIC YEAR
FIRST YEAR FIRST SEMESTER

SCHOOL OF SCIENCE AND INFORMATION SCIENCES
MASTER OF SCIENCE (APPLIED STATISTICS)

COURSE CODE: STA 8103
COURSE TITLE: MEASURE AND PROBABILITY THEORY

DATE: 2/2/2024 **TIME: 1430-1730 HRS**

INSTRUCTIONS TO CANDIDATES

i. Question **ONE** is compulsory
ii. Answer any other **TWO** questions

QUESTION ONE (20MARKS)

a. Define the following terms

i. ỏ-field (2marks)

ii. Burel-field (2marks)

b. Proof that a ỏ –field is a monotone field and conversely (4marks)

c. What is meant by the term indicator function of a set A (2marks)

d. Proof the following properties of indicator functions:

i. If ACB, then $I_A \leq I_B$ (4marks)

ii. $I_{(AUB)} = I_A + I_B - I_{AB}$ (2marks)

iii. $I_A{}^C = 1 - I_A$ (2marks)

iv. $I_{AB} = I_A I_B$ (2marks)

QUESTION TWO (20 MARKS)

Let (Ω, Ƒ, μ) be a measure state and let $(f_i)_i{}^{\infty}=1$be measurable functions from **Ω** to R such that $f_i\uparrow$ f a.e and $\int f_1 d\mu > -\infty$, then $\int f_i d\mu \uparrow \int f d\mu$.

Proof (20marks)

QUESTION THREE (20MARKS)

a. Giving examples distinguish between

i. Convergence almost strictly (4marks)

ii. Convergence in probability (4marks)

iii. Convergence in L^p (4marks)

iv. Convergence in L^q (4marks)

b. State the Burel _Catelli Lemma (4marks)

QUESTION FOUR (20MARKS)

a. Let f be a non-negative measurable function and t>0. Then (f>t) = [wϵΩ: f(w) > t] $\mu(\{f>t\}) \leq t^{-1} \int fd_u$.
   Proof (4marks)

b. Let (X,x,μ) and (Y,y,v) be finite measure spaces and let

F = {ECXxY:$\iint 1_E(x,y)d\mu(x)dv(y)=\iint 1_E(x,y)dv(y)d\mu(x)$ then XxyCF.

Proof (6marks)

c. Let F:R→R be non-constant, right continuous, and a non-decreasing dF(a,b) = F(b)-F(a).

Proof (10marks)

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