# MAASAI MARA UNIVERSITY

## REGULAR UNIVERSITY EXAMINATIONS

## 2023/2024 ACADEMIC YEAR

## FIRST YEAR FIRST SEMESTER

## SCHOOL OF PURE APPLIED AND HEALTH SCIENCES

## MASTER OF SCIENCE IN PURE MATHEMATICS

## COURSE CODE: MAT 8105

## COURSE TITLE: ABSTRACT INTEGRATION

**DATE:** **DURATION:**

**INSTRUCTIONS TO CANDIDATES**

Answer Question **ONE** and any other **TWO** questions

*This paper consists of **THREE** printed pages. Please turn over.*

## QUESTION ONE (30 MARKS)

**a)** Let A and B be non-empty sets, prove monotonicity of the outer measure on A and B. **(4 marks)**

**b)** Prove that if E has measure zero, then every subset of E is measurable. **(5 marks)**

**c)** Define a measurable function hence prove that a constant function with measurable domain is measurable. **(3 marks)**

**d)** State and prove monotone convergence theorem. **(5 marks)**

**e)** Using counter example, show that bounded convergence theory need not be true in Riemann integral. **(4 marks)**

**f)** Define a Cauchy sequence in measure hence show that if a sequence $\langle f_n \rangle$ convergence in measure to f, then $\langle f_n \rangle$ is a Cauchy sequence in measure. **(4 marks)**

**g)** Using a counter example show that a bounded measurable function need not be Riemann integrable. **(5 marks)**

## QUESTION TWO (15 MARKS)

**a)** Show that the outer measure is translational invariant. **(3 marks)**

**b)** Define convergence of measurable functions $\langle f_n \rangle$ hence show that if a sequence $\langle f_n \rangle$ converges in measure to a function f, then the limit function f is unique almost everywhere $\langle f_n \rangle$. **(4 marks)**

**c)** State and Prove Fatou's lemma. **(4 marks)**

**d)** Prove that if f is a measurable function over set E and if g is integrable function such that $|f| \le g$, the f is integrable over E. **(4 marks)**

## QUESTION THREE (15 MARKS)

**a)** Prove that the union of two measurable sets is measurable. **(5 marks)**

**b)** Define a σ-algebra hence show that the collection M of measurable sets is a σ-algebra. **(5 marks)**

**c)** Let f be a bounded function defined on [a,b], show that if f is Reimann integrable on [a,b] then it is Lebesque measurable and $R\int_a^b f(x)dx = \int_a^b f(x)$ .**(5 marks)**

**QUESTION FOUR (15 MARKS)**

a) Prove that every borel set is measurable. **(3 marks)**

b) State and prove Lebesque bounded convergence theorem. **(5 marks)**

c) Prove that $f = 0$ a.e if $\int_E f = 0$ and $f(x) \geq 0$ on E. **(7 marks)**