MAASAI MARA UNIVERSITY

SCHOOL OF BUSINESS AND ECONOMICS

REGULAR UNIVERSITY EXAMINATIONS
2023/2024 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER EXAMINATION
FOR THE
DEGREE OF BACHELOR OF MASTER SCIENCE IN ECONOMICS AND STATISTICS

COURSE CODE: ECS 8204
COURSE TITLE: TEST OF HYPOTHESIS

Date: JANUARY 2024 **Time: 3 HOURS**

INSTRUCTIONS TO CANDIDATES

*Answer question **ONE (compulsory)** and any other **TWO** questions.*

QUESTION ONE (30 MARKS)

a) Define the following terms:

   i) Type I error [1 mark]

   ii) Type II error [1 mark]

   iii) Most powerful test [2 mark]

b) Suppose we want to carry out the following hypothesis test for the mean, μ, of a normal distribution, with known variance, $\sigma^2 = 125$; H_0: $\mu = 100$ and H_1: $\mu > 100$. Suppose we decide to take a sample of size 36 and the level of significance, at α=0.05.

   i) What is the critical region? [2 marks]

   ii) What is the power of the test at $\mu = 110$? [2 marks]

c) In Comparing the variability of the tensile strength of two kinds of of structural steel, an experiment yielded the following results: $m = 13, n = 16$, the sample variance of the first sample was $S_1^2 = 19.2, S_2^2 = 3.5$. Assuming that the measurement constitutes an independent random sample from normal population. Test the null hypothesis that:

$H_0: \sigma_1^2 = \sigma_2^2$ Against $H_1: \sigma_1^2 \neq \sigma_2^2$ at $\alpha = 0.05$ [5 marks]

d) Suppose that the distribution of lifetimes of TV tubes can be modelled by an exponential distribution with mean θ. So $f(x,\theta) = \begin{cases} \frac{1}{\theta} e^{\frac{x}{\theta}} & for\ x \geq 0 \\ 0 & Otherwise \end{cases}$

Under usual conditions, the mean lifetime is 2000hrs but if a fault occurs, the mean lifetime drops to 1000hrs. A random sample of 20 tubes lifetime is taken in order to test the hypothesis $H_0: \theta = 2000$ against $H_1: \theta = 1000$. Use Neyman-Pearson lemma to find the Most Powerful test with α level of significance.

[5 marks]

e) A food processing company packages honey in glass jars. Each jar is supposed to contain 10 litres of honey. Previous experience suggests that the volume X, of a randomly selected jar of the company's honey is normally distributed with a known variance of 0.02. Derive the generalized likelihood ratio test for testing, at a significance level of $\alpha = 0.01$, the null hypothesis $H_0: \mu = 10$ against the alternative hypothesis $H_1: \mu \neq 10$ (6 marks)

f) Four different brands of margarine were analyzed to determine the level of some unsaturated fatty acids. The data are shown below. Perform an appropriate non parametric test at $\alpha = 0.05$. (6 marks)

Brand	Fatty Acids (%)				
A	13.5	13.4	14.1	14.2	
B	13.2	12.7	12.6	13.9	
C	16.8	17.2	16.4	17.3	18.0
D	18.1	17.2	18.7	18.4	

QUESTION TWO (20 MARKS)

a) Let $(x_1, y_1), (x_2, y_2) \ldots\ldots.. (x_n, y_n)$ be a random sample of size n from the bivariate normal population. Derive the test statistic for testing the hypothesis H_0: $\ell = 0$ against H_1: $\ell \neq 0$, where ℓ is the correlation co-efficient between X and Y. [12 marks]

b) Use the test statistic derived in (a) to test the hypothesis H_0: $\ell = 0$ against H_1: $\ell \neq 0$ at $\alpha = 5\%$ if the observations are (33, 24), (60, 34), (19, 64) (19, 24), (39, 34), assuming this sample is drawn from a bivariate normal population. [8 marks]

QUESTION THREE (20 MARKS)

a) Let X and Y be two independently distributed random variables with distributions X ~N $[\mu_{1,}\sigma_1^2]$ and Y~ $[\mu_2, \sigma_2{}^2]$ respectively.
Let $x_1\ x_2 \ldots.. x_m$ be a random sample of size m from X and $y_1, y_2, \ldots.., x_n$ be Another independent random sample of size n from Y.

Derive the likelihood ratio test for testing H_0: $\mu_2 = \mu_1$ against H_1: $\mu_1 \neq \mu_2$, Assuming $\sigma_1{}^2 = \sigma_2{}^2 = \sigma^2$ [16 marks]

b) The following statistics were obtained from data drawn from two independent populations X and Y which are normally distributed as follows: X ~N $[\mu_1, \sigma_m^2]$ and Y~ $[\mu_2, \sigma_n^2]$

$$\overline{X} = 1.02, \quad \sum_{i=1}^{m}(X_1 - \overline{X})^2 = 2.44, \text{ m = 11}$$

$$\overline{Y} = 1.66, \quad \sum_{i=1}^{n}(Y_1 - \overline{Y})^2 = 4.23, \text{ n = 13}$$

Test H_0: $\mu_1 = \mu_2$ against H_1: $\mu_1 \neq \mu_2$. Use $\propto = 5\%$ [4 marks]

QUESTION FOUR (20 MARKS)

a) State Neyman-Person Lemma for testing a simple hypothesis against simple alternative hypothesis. [4 marks]

b) Let $x_1, x_2, \ldots. X_n$ be a random sample from a normal variable X with mean μ and variance σ^2, where both μ and σ^2 are unknown.
Derive the test statistic for testing H_0: $\sigma^2 = \sigma_0{}^2$, against H_1: $\sigma^2 \neq \sigma_0{}^2$.
Use $\propto$ % [10 marks]

c) A random sample of n = 7 observations from a normal population produced the following measurements: 4, 0, 6, 3, 3, 2, 5, 9. Do the data provide sufficient evidence to indicate that $\sigma^2 > 1$? Test using $\propto = 0.05$ [6 marks]

QUESTION FIVE (20 MARKS)

a) (i) Let (x_1, y_1), (x_2, y_2) …….. (x_n, y_n) be a random sample of size n from the bivariate normal population. Derive the test statistic for testing the hypothesis H_0: $\ell = 0$ against H_1: $\ell \neq 0$, where ℓ is the correlation co-efficient between X and Y. [9 marks]

(ii) Use the test statistic derived in (a) to test the hypothesis H_0: $\ell = 0$ against H_1: $\ell \neq 0$ at $\propto = 5\%$ if the observations are (33, 24), (60, 34), (19, 64) (19, 24), (39, 34), assuming this sample is drawn from a bivariate normal population. [5 marks]

b) Use a Mann-Whitney U test to test if heart rate differs between men and women at the 95% level of significance. [6 marks]

Heart rate women (bmp)	Heart rate men (bmp)
84	80
81	74
80	73
70	72
72	78
69	75
65	70
74	74
80	69