MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2023/ 2024 ACADEMIC YEAR
THIRD YEAR FIRST SEMESTER

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES
BACHELOR OF SCIENCE IN APPLIED STATISTICS WITH COMPUTING

COURSE CODE: STA 4135-1

COURSE TITLE: APPLIED TIME SERIES ANALYSIS

DATE: 5/12/2023 **TIME: 1100-1300 HRS**

INSTRUCTIONS TO CANDIDATES

Answer Question ONE and any other TWO questions

Question One (20 Marks)

a) Name and briefly describe the objectives of time series **(3 Marks)**

b) Consider the following second order autoregressive process

$X_t = X_{t-1} - \frac{1}{2} X_{t-2} + e_t$ where $\{e_t\}$ is the white noise process.

(i) Is this process stationary? (2 Marks)

c) The following data was obtained for a process Z_t

t	1	2	3	4	5	6	7	8	9	10
Z_t	13	8	15	4	4	12	11	7	14	12

Find the sample autocovariances r(1) and r(4) and the sample autocorrelations $\hat{\rho}_1$ and $\hat{\rho}_4$ (**6 Marks)**

d) Given that $\hat{\rho}_1 = -0.188$, $\hat{\rho}_2 = -0.201$, $\hat{\rho}_3 = 0.181$. Find $\hat{\phi}_{11}$, $\hat{\phi}_{22}$ *and* $\hat{\phi}_{33}$ **(5 Marks)**

e) Find the spectral density function of the process X_t given by

$X_t = 0.2X_{t-1} + e_t$ **(4 Marks)**

Question Two (15 Marks)

a) What do you understand by the following statements?

(i) Time series is stationary in the weak sense

(ii) Time series is stationary in the strict sense **(3 Marks)**

b) Let a mixed autoregressive moving average process be given by

$X_t = X_{t-1} - \frac{3}{16} X_{t-2} + e_t + \frac{1}{2} e_{t-1} + \frac{1}{4} e_{t-2}$.

Is the process stationary and invertible? **(5 Marks)**

c) Find the spectral density function of the process X_t given by

$X_t = e_t - 2e_{t-1} + e_{t-2}$ **(7 Marks)**

Question Three (15 Marks)

a) Describe the components of time series. **(4 Marks)**

b) Show that the ACF of ARMA(1, 1) process

$X_t = \alpha X_{t-1} + e_t + \beta e_{t-1}$ is given by

$$\rho(1) = \left(\frac{(1+\alpha\beta)(\alpha+\beta)}{1+\beta^2+2\alpha\beta} \right)$$

$$\rho(h) = \alpha\rho(h-1)$$

Assuming that $\{X_{t-1}\}$ be stationary process and $E(X_t) = 0$ **(11 Marks)**

Question Four (15 Marks)

a) Define the following terms:

(i) the autocorrelation function (ACF) of stationary stochastic process in determining the order of a moving average process

(ii) the white noise process **(5 Marks)**

b) The following data was obtained for a process X_t

t	1	2	3	4	5	6	7
X_t	30.50	29.83	20.48	26.26	29.50	31.53	33.18

Find $\hat{\phi}_{11}$, $\hat{\phi}_{22}$ *and* $\hat{\phi}_{33}$ **(6 Marks)**

c) Find the PACF for the following processes

(i) $X_t = 0.8X_{t-1} + e_t$

(ii) $X_t = 0.3X_{t-1} + 0.6X_{t-2} + e_t$ **(4 Marks)**

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