MAASAI MARA UNIVERSITY

UNIVERSITY EXAMINATIONS 2022/2023

THIRD YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN APPLIED STATISTICS WITH COMPUTING AND BACHELOR OF SCIENCE IN MATHEMATICS

STA 2221-1: OPERATION RESEARCH II

DATE: APRIL 2023 **TIME: 2 HOURS**

INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE (20MARKS)

a) Define the following terms as used in operation research and decision making theory

i. Admissibility (1 mark)
ii. Minimax risk (1 mark)
iii. Baye's risk (1 mark)

b) Consider the payoff table below

	Bad sales	Good sales
	E1	E_2
New machine d_1	320	460
Overtime d_2	370	400
Prob	0.35	0.65

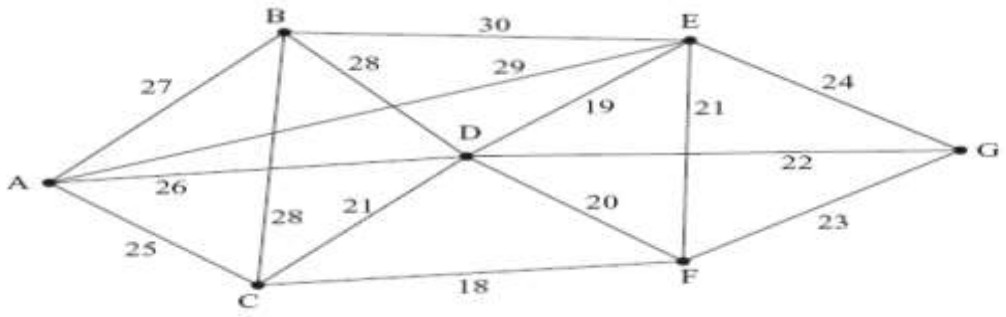
i. Compute the expected money value and determine the optimum decision (2 marks)
ii. Construct a decision tree for the problem (1 mark)
iii. How critical is the choice of probability for sale to be good is 0.65 (1 mark)
iv. Compute the expected money value under uncertainty (1 mark)
v. Determine expected value of perfect information (1 mark)

c) State the difference between Kruskal's aligorithm and Prim's aligorithm for finding a minimum spanning tree (2 Marks)

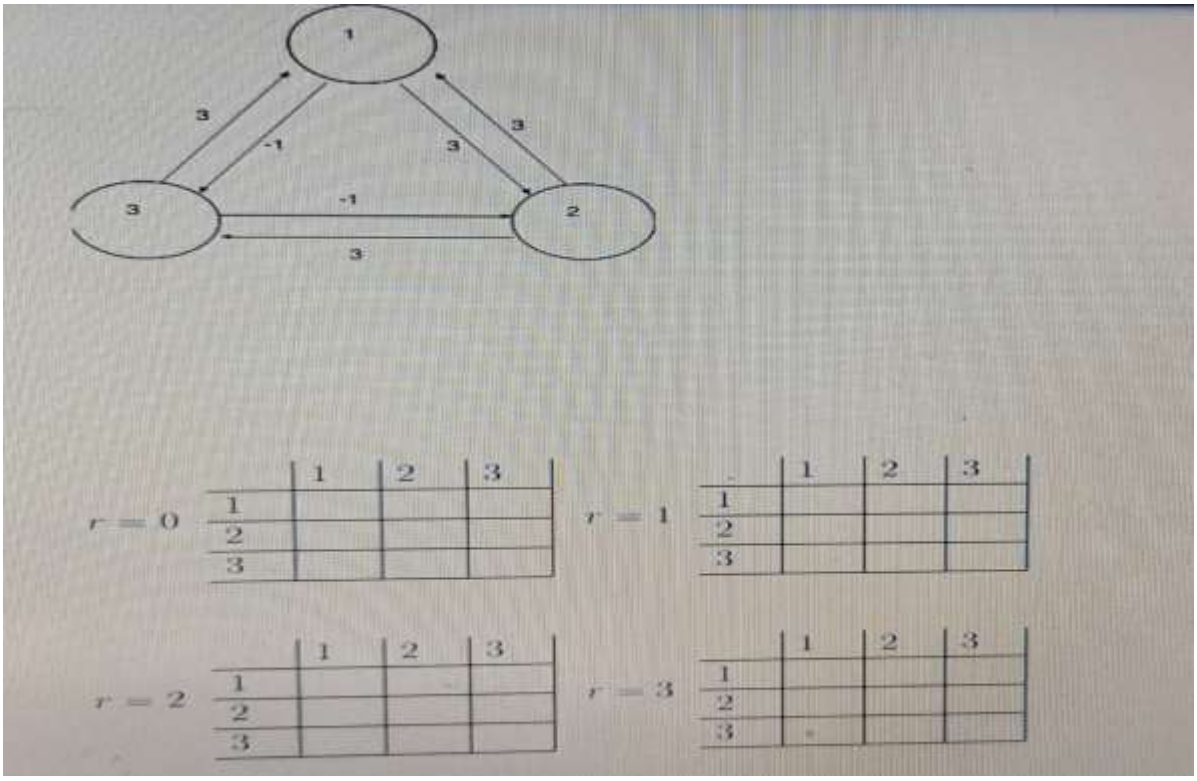
d) Listing the areas in the order that you consider them, find a minimum spanning tree for the network spanning tree, using

i) Prim's aligorithm (2 Marks)

ii) Kruskal's aligorithm (2 Marks)



e) Recall that in the Floyd-Warshall algorithm dist(u, v, r) is defined to be the shortest path from u to v where all intermediate vertices (if any) are numbered r or less. For the following graph, fill in the distance arrays computed by Floyd-Warshall for all values of r. In the distance arrays, let the row be the vertex the path starts at and let the column be the vertex the path ends at. (5 Marks)



QUESTION TWO (20 MARKS)

a) On a given day the weather condition is either rainy θ_1 or sunshine θ_2. An individual has the following options, stay home a_1 go out without umbrella a_2 or go out with umbrella a_3. Shown on the table below

	θ_1 Rainy	θ_2 Sunny
a_1 stay home	4	5
a_2 go out without umbrella	5	2
a_3 go out with umbrella	3	0

i. Assuming that probability that it rains is $\frac{1}{2}$. Compute the risk for all decisions and choose the optimum (2 marks)

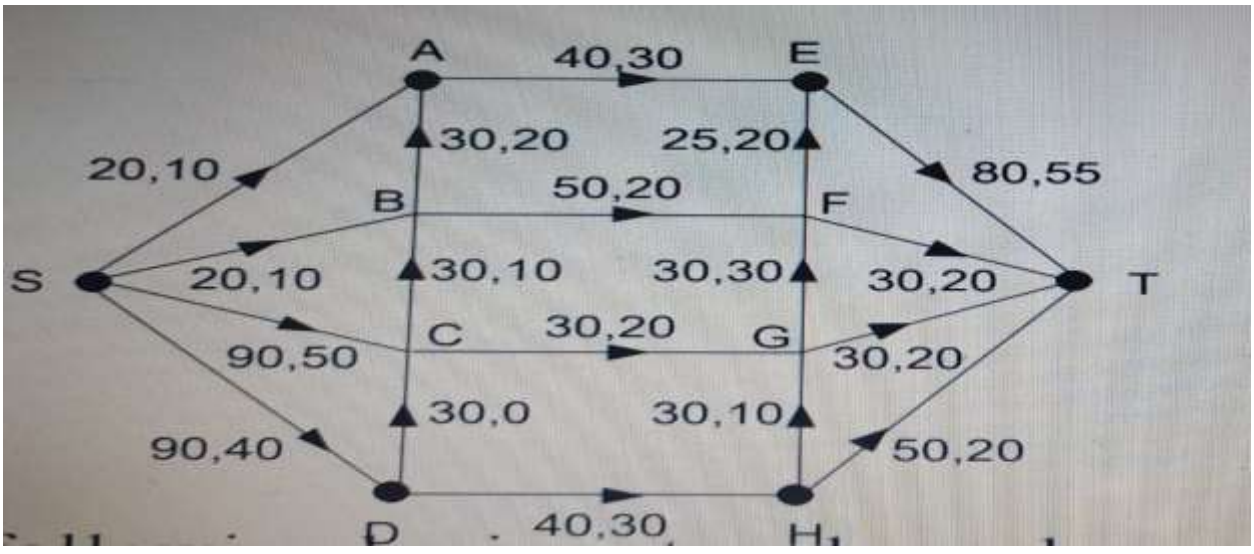
ii. Additional information is provided by weather forecast $(Y = y)$ with conditional probability as follows:

$$p(Y = y/\theta = 0.6 \qquad p(Y \neq y/\theta_2) = 0.8$$
$$p(Y \neq y/\theta = 0.4 \qquad p(Y = y/\theta_2) = 0.2$$

Compute the posterior distribution

$$p(\theta = \theta/Y = y) \quad \text{and} \quad p(\theta{:}\,\theta_2/Y = y) \qquad \text{(3 marks)}$$

b) The network below shows the maximum and minimum flow allowed along each arc network.



i) Ignoring the minimum flow constraints, find a feasible flow between S and T of value 140. (3 marks)

ii) Find the maximum flow, when both maximum and minimum constraints operate. Explain why your flow is a maximum flow (3 marks)

c) Busy Transport Company records that the arrival of tracks carrying goods is.30 per day. Assuming the interarrival time follows an exponential distribution and the service time is also an exponential with an average of 36 minutes. Calculate:

i. The expected queue size (1 Mark)

ii. Probability that the queue size exceeds 10 (1 Mark)

iii. If the input of tracks increases to an average of 33 per day, what will be change in (i) and (ii) (2 Marks)

QUESTION THREE (15 MARKS)

a) Listed in the table below are activities and sequencing necessary for maintenance of job on heat exchangers in refinery. Draw network diagram of the project. (5 Marks)

Activity	Description	Predecessor Activity
A	Dismantle pipe connection	A
B	Dismantle heater, closure, and floating front	B
C	Remove the tube bundle	B
D	Clean tube bundle	C
E	Clean shell	C
F	Replace tube bundle	F,G
G	Prepare shell pressure test	D,E,H
H	Prepare tube pressure test and reassemble	G

b) A small project is composed of 7 activities whose time estimates in weeks are listed below.

Activity	Predecessors	Optimistic	Most likely	Pessimistic
A	-	1	2	4
B	-	5	6	7
C	-	2	4	5
D	A	1	3	4
E	C	4	5	7
F	A	3	4	5
G	B,D,E	1	2	3

i. Draw the network (3 marks)
ii. Calculate the expected project duration and the variance of the project duration based on network analysis. (3 marks)
iii. Find the expected project completion time (1 marks)
iv. Calculate the probability that the project will be completed on or before a deadline of 10 weeks. (3 marks)

QUESTION FOUR (15 MARKS)

a) A business man has three alternatives open to him each of which can be followed by any of the four possible events. The conditional pay offs for each action - event combination are given below:

Alternative	Pay – offs (Conditional events)			
	A	*B*	*C*	*D*
X	8	0	–10	6
Y	–4	12	18	–2
Z	14	6	0	8

Determine which alternative the businessman should choose, if he adopts the maximin principle. (4 marks)

b) Consider the following pay-off matrix

Alternative	Pay – offs (Conditional events)			
	A_1	A_2	A_3	A_4
E_1	7	12	20	27
E_2	10	9	10	25
E_3	23	20	14	23
E_4	32	24	21	17

Using minmax principle, determine the best alternative. (4 marks)

c) The advertising alternatives for a company include television, radio, and newspaper advertisements. The costs and estimates for audience coverage are given in the table below.

	Television	Newspaper	Radio
Cost per advertisement	$2,000	$600	$300
Audience per advertisement	100,000	40,000	18,000

The local newspaper limits the number of weekly advertisements from a single company to ten. Moreover, in order to balance the advertising among the three types of media, no more than half of the total number of advertisements should occur on the radio, and at least 10% should occur on television. The weekly advertising budget is $18,200. How many advertisements should be run in each of the three types of media to maximize the total audience? Use the simplex method to solve this problem.

(7 marks)