MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS

2022/2023 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES

BACHELOR OF SCIENCE (APPLIED MATHEMATICS)

COURSE CODE: MAT 4242

COURSE TITLE: ANALYTICAL MECHANICS

DATE: 20/4/ 2023   **TIME: 1430-1630**

INSTRUCTIONS TO CANDIDATES

1. Answer **Question One (Compulsory)** and **ANY** other **Two Questions.**
2. All Examination Rules Apply.

This paper consists of 3 printed pages. Please turn over

Question One (30 Marks)

a. Define a central force **(2 Marks)**

b. An xyz coordinate system is rotating w.r.t XYZ coordinate system having the same origin and assumed to fixed in space i.e an inertial system. The angular velocity of the xyz system relative to the XYZ system is given by $\vec{w} = 2ti - t^2 j + (2t+4)k$ where t is the time. The position vector of a particle at time t as observed in the xyz system is given by $\vec{r} = (t^2+1)i - 6tj + 4t^3k$ .Find

    i) The apparent velocity and true velocity at time $t = 1$ **(3 Marks)**

    ii) The apparent acceleration and the true acceleration at time $t = 1$ **(3Marks)**

    iii) The Coriolis acceleration and its magnitude at $t = 1$ **(3 Marks)**

c. State and explain three properties of central force fields **(6 Marks)**

d. Prove that if a particle moves in a central force field, then its path must be a plane curve **(4 Marks)**

e. Consider, a simple pendulum of mass m and on a string of length L. obtain the equation of motion of this simple pendulum using the Lagrange equation. **(5 Marks)**

Question Two (20 Marks)

a. State the three Kepler's laws **(6 Marks)**

b. A particle moves under a central force field and describes the path $r^n = a^n \cos n\theta$, Show that the force field is proportional to $\dfrac{1}{r^{2n+3}}$ **(8 Marks)**

c. A canon whose mass is 1300kg fires a ball in the horizontal direction at a speed of 40m/s relative to the cannon. If the cannon is mounted and can record freely. What is the recoil velocity of the cannon with respect to the Earth?
What is the initial velocity of the ball with respect to the Earth? **(6 Marks)**

Question Three (20 Marks)

a. State two properties an isolated two body system   **(2 Marks)**
b. Define a rigid body and explain the meaning of degrees of freedom
                                                                        (4 Marks)
c. A particle of mass m moves along a trajectory given by $x = x_0 \cos w_1 t$,

   $y = y_0 \sin w_2 t$. Find the x and y components of the force stating the conditions under which the force is a central force.
                                                                        (5 Marks)
d. A rigid body is in space. All the external influences (including gravity) are negligible. Use the Newton's laws to show that the angular momentum is conserved   **(4 Marks)**
e. State and explain three characteristics of nonlinear oscillators   **(5 Marks)**

Question Four (20 Marks)

a. State the D'Alembert's principle   **(2 Marks)**

b. A rigid body is rotating with a constant angular speed of 3 radians per second about a fixed axis through the points A (4, 1, 1), B (2,-1, 0) with the distances being measured in cm. The rotation is in the left-handed sense relative to the direction $\overrightarrow{AB}$. Find the instantaneous velocity and acceleration of the particle P of the body at the point (4, 4, 4).   **(6 Marks)**
c. A mass m, is attached to a support by a spring constant k, the mass is hanging down from a spring so that there is a gravitational force on the mass as well. The support is oscillating with an amplitude of A and at a frequency of ωa. Neglecting any resistance or frictional forces, Find the motion of the mass relative to the support and the motion of the mass relative to the inertial frame
                                                                        (6 Marks)
d. Define Coriolis force and give an example where the force is experienced
                                                                        (2 Marks)
e. State the Hamiltonian principle and use it to find the equation of motion of a one-dimensional harmonic oscillator of mass m   **(4 Marks)**

//END//