MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR
THIRD YEAR SECOND SEMESTER

SCHOOL OF BUSINESS AND ECONOMICS
BACHELOR OF SCIENCE (ECONOMICS & STATISTICS)

COURSE CODE:ECS 3204
COURSE TITLE: THEORY OF ESTIMATION

DATE: 19/4/ 2023 **TIME: 1100-1300 HRS**

INSTRUCTIONS TO CANDIDATES

1. Answer **Question ONE** and any other **Two** questions.
2. Show all the workings clearly
3. Do not write on the question paper
4. All Examination Rules Apply.

QUESTION ONE (30 MARKS)

(a) Let $x_1, x_2, ..., x_n$ be a random sample from the p.d.f

$$f(x;\theta) = \begin{cases} \theta^2 x e^{-\theta x}, & 0 < x < \infty \\ 0 & , elsewhere \end{cases}$$

Find (i) the method of moments estimator for θ **(5 marks)**

(ii) the maximum likelihood estimator of θ **(3 marks)**

(b) (i) Explain what is meant by a sequence of estimators being mean squared error consistent **(3 marks)**

(ii) Define a $UMVUE$ for a parameter $\tau(\theta)$ **(4 marks)**

(c) Let $x_1, x_2, ..., x_n$ be a random sample from the p.d.f

$$f(x;\theta) = \begin{cases} \theta(1-\theta)^x, & x = 0,1,2,...,\ 0 < \theta < 1 \\ 0 & , elsewhere \end{cases}$$

Find the sufficient statistics for θ **(3 marks)**

(d) Let $x_1, x_2, ..., x_n$ be a random sample from $N(\mu, \sigma^2)$ where μ is known and σ^2 is unknown. Find the $UMVUE$ of σ^2 **(6 marks)**

(e) Independent random samples of the heights of adult males living in two counties yielded the following results.

$$n = 12, \quad \bar{x} = 65.7 \quad \sigma_x^2 = 16$$
$$m = 15, \quad \bar{y} = 68.2 \quad \sigma_y^2 = 9$$

Find the 98% confidence interval for the difference $\mu_x - \mu_y$ **(5 marks)**

QUESTION TWO (20 MARKS)

(a) Describe the method of maximum likelihood estimation. **(8 marks)**

(b) Let $x_1, x_2, ..., x_n$ be a random sample from the p.d.f

$$f(x;\theta) = \begin{cases} \theta x^{\theta-1}, & 0 < \theta < 1,\ 0 < \theta < \infty \\ 0 & , elsewhere \end{cases}$$

(i) Show that the maximum likelihood estimator of θ is $\hat{\theta} = -\frac{n}{\ln\left(\prod_{i=1}^{n} x_i\right)}$ **(3 marks)**

(ii) Below are 10 observations from this distribution

0.2228	0.7112	0.9924	0.9518	0.8609
0.8278	0.7464	0.4374	0.3125	0.9960

Find the maximum likelihood estimate of θ **(3 marks)**

(c) For the p.d.f in (b) above, find

(i) the method of moments estimator for θ

(ii) the moments estimate of θ **(6 marks)**

QUESTION THREE (20 MARKS)

(a) State the Cramer-Rao inequality **(3 marks)**

(b) Let $x_1, x_2, ..., x_n$ be a random sample from $N(\theta, 9)$

(i) Find the Cramer-Rao lower bound for the variance of unbiased estimators of θ **(5 marks)**

(ii) the $UMVUE$ of θ **(8 marks)**

(c) Find the $UMVUE$ of $\frac{1}{\theta}$ given that $x_1, x_2, ..., x_n$ be a random sample from

p.d.f $f(x;\theta) = \begin{cases} \theta e^{-\theta x}, & 0 < x < \infty,\ 0 < \theta < \infty \\ 0 & , elsewhere \end{cases}$ **(4 marks)**

QUESTION FOUR (20 MARKS)

(a) Define a pivotal quantity **(3 marks)**

(b) Let $x_1, x_2, ..., x_n$ be a random sample from $N(\mu, \sigma^2)$ where both μ and σ^2 is unknown.

Find the $100(1-\alpha)\%$ confidence interval for σ^2. **(8 marks)**

(c) Below are 25 observed values from $N(\mu, \sigma^2)$

232	318	174	208	380	308	277	315	248
.306	269	325	190	266	285	222	210	
258	183	172	215	228	241	252	144	

Find the 90% confidence interval for σ^2 **(9 marks)**

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