MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS

2023/2024

SCHOOL OF BUSINESS AND ECONOMICS

BACHELOR'S OF SCIENCE IN ECONOMICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER

COURSE CODE: ECS 4105-1

COURSE TITLE: THEORY OF ESTIMATION

DATE: **TIME:**

INSTRUCTIONS:

Attempt Question one and any other Two Questions

Show your workings as marks will be awarded for correct working.

Question One

a. Differentiate the following types of estimators
   i. Consistent and Unbiased. **(2 marks)**
   ii. Efficient and Sufficient. **(2 marks)**
   iii. Point and Interval. **(2 marks)**

b. Let $X_1, X_2, \ldots, X_n$ be random sample from an exponential distribution with parameter λ. Determine the method of moment estimator for λ. **(4 marks)**

c. For a geometric distribution with parameter p. Show that $T(x) = \sum x_i$ is a sufficient statistic for the parameter p. **(3 marks)**

d. For a normal distribution with mean μ and variance σ^2. Show that the sample mean $\overline{X}$ is weakly consistent estimator for the population mean μ. **(4 marks)**

e. A random sample 64 students were determined to have an average weight of 67Kg with a standard deviation of 5.2Kgs. Construct a 95% confidence interval for the true average weight for the students. **(3 marks)**

Question Two

Let $X_1, X_2, \ldots, X_n$ be random sample from a normal population with mean μ and variance σ^2.

a. Determine the MLE estimator for μ. **(4 marks)**

b. Show that the estimator in (a) is unbiased. **(3 marks)**

c. Determine the MLE estimator for σ^2. **(4 marks)**

d. Show that the estimator in (c) is MLE consistent. **(4 marks)**

Question Three

a. For a simple linear regression model $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$ the Ordinary Least Square estimator for the parameters β_0 is $\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1 \overline{X}$ while that for β_1 is $\hat{\beta}_1 = \dfrac{\sum (Y_i - \overline{Y})(X_i - \overline{X})}{\sum (X_i - \overline{X})^2}$. Show that if $\varepsilon_i \sim N(0, \sigma^2)$ then the MLE estimators for β_0 and β_1 are equal to those of the OLS. **(9 marks)**

b. Show that for a random sample $X_1, X_2, \ldots, X_n$ from a normal distribution with mean μ and variance σ^2. Show that the sample variance $S^2 = \dfrac{1}{n-1}\sum (X_i - \overline{X})^2$ is unbiased estimator for the population variance σ^2. **(6 marks)**

Question Four

a. Define an MVUE. **(2 marks)**

b. State the Rao-Blackwell theorem. **(2 marks)**

c. Show that the statistic $T(x)=\sum x$ is a minimal sufficient statistic for the parameter θ in a Bernoulli distribution. **(3 marks)**

d. A multiple linear regression model can be written in matrix for as $Y = X\beta + \varepsilon$ where Y is an $n \times 1$ matrix, β is a $k \times 1$ matrix, X is a $n \times k$ matrix and ε is a $n \times 1$ matrix such that $E(\varepsilon)=0$ and $\text{var}(\varepsilon)=\sigma^2$. Show that the OLS estimator is BLUE. **(8 marks)**