MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS

2023/2024

SCHOOL OF BUSINESS AND ECONOMICS

BACHELOR'S OF SCIENCE IN ECONOMICS AND STATISTICS

SECOND YEAR FIRST SEMESTER

COURSE CODE: ECS 2103-1

COURSE TITLE: BIVARIATE DISTRIBUTION

DATE: **TIME:**

INSTRUCTIONS:

Attempt Question one and any other Two Questions

Show all your workings

Question One

a. Random variables X and Y have a joint probability density function given as;

$$f(x,y)=\begin{cases} cxy & ,0<x<3, 0<y<3 \\ 0 & ,otherwise \end{cases}$$

Determine;

i. The value of c. **(2 marks)**

ii. $\Pr(X<2, Y>1)$. **(3 marks)**

iii. If X and Y are independent. **(4 marks)**

b. The data below shows a joint probability mass function between X and Y.

X	1	1	2	4
Y	3	4	5	6
$f(x,y)$	1/8	1/4	1/2	1/8

Determine $corr(x,y)$ and hence comment on the nature of relationship between X and Y.

(4 marks)

c. The data below shows the quantity demanded and price of cassava in a certain market.

Quantity Demanded in Kg (x)	Price in Shillings (y)
30	120
45	110
60	90
70	85
55	94
66	89
34	119
56	93
38	115
40	100

i. Fit a simple linear for price of cassava as a function of quantity demanded hence interpret the slope. **(5 marks)**

ii. Estimate the price of cassava in the market when the quantity demanded is 62 kg. **(1 mark)**

iii. Based on the fitted regression model does the interpretation of the intercept make sense? If yes, interpret it, if no, state why? **(1 mark)**

Question Two

a. X and Y have a joint probability mass function given as.

$$f(x,y)=\begin{cases} p^{x+y}(1-p)^{2-x-y} & x=0,1; y=0,1 \\ 0 & elsewhere \end{cases}$$

Determine;

i. $M_{xy}(t_1, t_2)$. **(3 marks)**

ii. $E(x)$ using $M_{xy}(t_1, t_2)$. **(2 marks)**

iii. $E(y)$ using $M_{xy}(t_1, t_2)$. **(2 marks)**

iv. $E(x, y)$ using $M_{xy}(t_1, t_2)$. **(2 marks)**

b. Two bivariate random variables X and Y have a variance covariance matrix.

$$\Sigma_{XY} = \begin{bmatrix} 1.5 & -1.4 \\ -1.4 & 2.1 \end{bmatrix}$$

Determine;

i. $\text{var}(3x + 4y - 1)$. **(2 marks)**

ii. $\text{var}(5x - y + 3)$. **(2 marks)**

iii. $corr(xy)$. **(2 marks)**

Question Three

Two continuous random variables have a joint probability density function.

$$f(xy) = \begin{cases} 2x + 2y - 4xy & 0 \leq x \leq 1; 0 \leq y \leq 1 \\ 0 & otherwise \end{cases}$$

Determine;

a. $f_x(x)$. **(2 marks)**

b. $f_y(y)$. **(2 marks)**

c. $corr(xy)$**.** **(7 marks)**

d. $\Pr(x > 0.3 \mid y = 0.5)$. **(4 marks)**

Question Four

a. A random variable X and Y have a joint probability mass function given as;

$$f(x, y) = \begin{cases} cxy & , x = 1,2,3; y = 1,2,3 \\ 0 & , otherwise \end{cases}$$

Determine;

i. The value of c. **(1 marks)**

ii. $M_{xy}(t_1, t_2)$. **(2 marks)**

iii. $E(x, y)$ using $M_{xy}(t_1, t_2)$. **(2 marks)**

c. Two random variables X and Y have a bivariate normal distribution with parameters $\sigma_x = 4, \sigma_y = 1, \mu_x = 2, \mu_y = 1$ and $\rho = -0.8$. Determine

i. The covariance between X and Y. **(2 marks)**

ii. $\Pr(x > 3)$. **(2 marks)**

iii. The conditional distribution of X given Y = 0.6. **(3 marks)**

iv. $\Pr\left(x > 3 \mid y = 0.6\right)$. **(3 marks)**