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Residual Reduced Balanced Incomplete Block Design

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Abstract: The construction of Balanced Incomplete Block Designs is a combination problem that involves the arrangement of v treatments into b blocks each of size k such that each treatment is replicated exactly r times in the design and a pair of treatments occur together in λ blocks. Researchers have devised a number of methods that can be used in constructing BIBDs, using geometry, difference sets, existing BIBD designs, computers and mathematical algorithms, and Latin squares. However, the existing constructing methods still cannot be used to construct all the BIBDs. This has left the existence of some BIBDs to still be unknown as some of them still cannot be constructed using the existing construction methods. The study aimed to derive a new construction method that uses the un-reduced BIBD to construct a new class of BIBD known as Residual Reduced BIBD. The study used the un-reduced BIBD with parameters (v, k) to construct the new class of BIBD. Consider an un-reduced BIBD with parameters v and k such that $k \geq 3$ the Residual Reduced BIBD was derived from the un-reduced design selection of blocks of the un-reduced BIBD that contain a particular treatment i. Then in the selected blocks if treatments i deleted and the rest of the treatments are left, then this forms a BIBD known as Residual Reduced BIBD. Residual Reduced BIBD formed has the parameters $v^* = v - 1$, $b^* = ((v-1)!(v-k))/(k!(v-k)!)$, $k^* = k$, $r^* = ((v-2)!(v-k))/((k-1)!(v-k)!)$, $\lambda^* = ((v-3)!(v-k))/((k-2)!(v-k)!)$. In conclusion, the study was able to show that a new class of BIBD could be constructed from the un-reduced BIBD. This means that some other BIBDs still can be derived from this universal set using appropriate procedures.

Keywords: Un-Reduced BIBD, Incomplete Block Design, Treatment, Reduced Residual, Balanced Incomplete Block Design, Construction of BIBD

1. Introduction

A Balanced Incomplete Block Design (BIBD) refers to the arrangement of v treatments into b blocks of size k each such that each treatment occurs r times in the entire design and each pair of treatment occur together λ number of times [17, 19-22].

Given this set of design arrangements falls squarely on combinatoric designs, a number of construction techniques have over the years been derived for constructing these designs. Most of these methods were first introduced by and over the years other researchers have tried to develop further other methods of constructing the designs. From the literature, a BIBD can be constructed using Projection Geometry, Euclidean Geometry, Cyclic Difference Sets Method, Symmetric Repeated Difference Method, Latin Square Method, Linear Integer Programming, Use of IBD package in R software, and Using Existing BIBD Designs [1, 4, 15, 16, 18, 21].

On the part of using existing BIBDs to construct new classes of BIBDs, a number of classes of BIBDs such as Derived BIBD, Residual BIBD, Complementary BIBD, and Dual BIBD have been discovered [2, 12]. For the case of Residual BIBD, using existing BIBD with parameters (v, b, r, k, λ), a Residual BIBD with parameters $v^* = v - k, b^* = b - 1, r^* = r, k^* = k - \lambda, \lambda^* = \lambda$ can be constructed if a block from the BIBD is deleted and then in the remaining $b - 1$ blocks all the treatments that were contained in the deleted block are deleted [2, 12].

Among all classes of BIBDs, a universal class of BIBD known as the Un-reduced BIBD exist. This class of BIBD is known to be created from a set of all possible combinations $\binom{v}{k}$. The design is believed to contain all other BIBDs with parameters v and k, hence it is considered as a universal set of BIBDs with parameters v and k. This class of BIBD forms BIBD with parameters $v^* = v, b^* = \binom{v}{k}, r^* = \binom{v-1}{k-1}, k^* = k, and\ \lambda^* = \binom{v-2}{k-2}$ [1-6, 10-12, 14].

Extensive work has been done over the years by various scholars on the construction of BIBDs. However, they have not exhausted all the available techniques for constructing BIBDs which leaves the existence of some BIBDs unknown because they still cannot be constructed using the existing methods [7-9, 13]. The present study intended to bridge this gap by introducing a new class of BIBD known as Residual Reduced Balanced Incomplete Block Design (RRBIBD) which uses the un-reduced BIBD to construct a new class of BIBD. The idea behind the technique was that given that un-reduced BIBD was a universal set of several classes of BIBD, new classes of BIBD which are still unknown may still exist within the design.

2. Residual Reduced BIBD

Theorem 1. Consider an un-reduced BIBD with parameters v and $k \geq 3$. When all the blocks that contain treatment i are removed from the design then the remaining blocks form a BIBD with parameters $v^* = v-1, b^* = \binom{v}{k} - \binom{v-1}{k-1} = \frac{(v-1)!(v-k)}{k!(v-k)!}, k^* = k, r^* = \binom{v-1}{k-1} - \binom{v-2}{k-2} = \frac{(v-2)!(v-k)}{(k-1)!(v-k)!}, \lambda^* = \binom{v-2}{k-2} - \binom{v-3}{k-3} = \frac{(v-3)!(v-k)}{(k-2)!(v-k)!}$.

Proof

We need to prove that the Residual Reduced BIBD satisfies the necessary conditions for the existence of a BIBD. First, we begin with the first condition.

$$\begin{aligned} b^*k^* &= \left(\frac{(v-1)!(v-k)}{k!(v-k)!}\right)k \\ &= \frac{(v-1)!(v-k)}{(k-1)!(v-k)!} \\ &= \left(\frac{(v-2)!(v-k)}{(k-1)!(v-k)!}\right)v-1 \\ &= r^*v^* \end{aligned}$$

The results show that the first condition of BIBD is satisfied. Next, we check the second condition for the existence of BIBD.

$$\begin{aligned} \lambda^*(v^*-1) &= \left(\frac{(v-3)!(v-k)}{(k-2)!(v-k)!}\right)(v-2) \\ &= \frac{(v-2)!(v-k)}{(k-2)!(v-k)!} \\ &= \left(\frac{(v-2)!(v-k)}{(k-1)!(v-k)!}\right)(k-1) \\ &= r^*(k^*-1) \end{aligned}$$

The results show that the second condition of BIBD is satisfied therefore, therefore, Residual Reduced BIBD is indeed a BIBD.

Example 1

Let's consider an un-reduced BIBD generated $v = 6$ treatments and $k = 3$ plots as shown in table 1.

Table 1. *Un-reduced BIBD with $v = 6$ and $k = 3$.*

Block 1 = {1,2,3}	Block 11 = {2,3,4}
Block 2 = {1,2,4}	Block 12 = {2,3,5}
Block 3 = {1,2,5}	Block 13 = {2,3,6}
Block 4 = {1,2,6}	Block 14 = {2,4,5}
Block 5 = {1,3,4}	Block 15 = {2,4,6}
Block 6 = {1,3,5}	Block 16 = {2,5,6}
Block 7 = {1,3,6}	Block 17 = {3,4,5}
Block 8 = {1,4,5}	Block 18 = {3,4,6}
Block 9 = {1,4,6}	Block 19 = {3,5,6}
Block 10 = {1,5,6}	Block 20 = {4,5,6}

From the BIBD in table 1 if we delete all the blocks that contain treatment 1 then then the BIBD illustrated in table 2 will result.

Table 2. *Residual Reduced BIBD with $v = 5, k = 3, b = 10, r = 6, \lambda = 3$.*

Block 1 = {2,3,4}
Block 2 = {2,3,5}
Block 3 = {2,3,6}
Block 4 = {2,4,5}
Block 5 = {2,4,6}
Block 6 = {2,5,6}
Block 7 = {3,4,5}
Block 8 = {3,4,6}
Block 9 = {3,5,6}
Block 10 = {4,5,6}

Example 2

Let's consider an un-reduced BIBD generated $v = 7$ treatments and $k = 5$ plots as shown in table 3.

Table 3. *Un-reduced BIBD with $v = 7$ and $k = 5$.*

Block 1 = {1,2,3,4,5}	Block 11 = {1,3,4,5,6}
Block 2 = {1,2,3,4,6}	Block 12 = {1,3,4,5,7}
Block 3 = {1,2,3,4,7}	Block 13 = {1,3,4,6,7}
Block 4 = {1,2,3,5,6}	Block 14 = {1,3,5,6,7}
Block 5 = {1,2,3,5,7}	Block 15 = {1,4,5,6,7}
Block 6 = {1,2,3,6,7}	Block 16 = {2,3,4,5,6}
Block 7 = {1,2,4,5,6}	Block 17 = {2,3,4,5,7}
Block 8 = {1,2,4,5,7}	Block 18 = {2,3,4,6,7}
Block 9 = {1,2,4,6,7}	Block 19 = {2,3,5,6,7}
Block 10 = {1,2,5,6,7}	Block 20 = {2,4,5,6,7}
	Block 21 = {3,4,5,6,7}

From the BIBD in table 3 if we delete all blocks that contain treatment 3 from the design then the BIBD illustrated in table 4 will result.

Table 4. *Residual Reduced BIBD with $v = 6, k = 5, b = 6, r = 5, \lambda = 4$.*

Block 1 = {1,4,5,6,7}
Block 2 = {1,2,4,5,6}
Block 3 = {1,2,4,5,7}
Block 4 = {1,2,4,6,7}
Block 5 = {1,2,5,6,7}
Block 6 = {2,4,5,6,7}

In general, the Residual Reduced BIBD technique can be used to construct the following number of BIBDs as illustrated in table 5.

Table 5. List of some Residual Reduced BIBD.

No.	Un-reduced BIBD	Residual Reduced BIBD
1	$v = 5, k = 3$	$v = 4, b = 4, r = 3, k = 3, \lambda = 2$
2	$v = 6, k = 3$	$v = 5, b = 10, r = 6, k = 3, \lambda = 3$
3	$v = 6, k = 4$	$v = 5, b = 5, r = 4, k = 4, \lambda = 3$
4	$v = 7, k = 3$	$v = 6, b = 20, r = 10, k = 3, \lambda = 4$
5	$v = 7, k = 4$	$v = 6, b = 15, r = 10, k = 4, \lambda = 6$
6	$v = 7, k = 5$	$v = 6, b = 6, r = 5, k = 5, \lambda = 4$
7	$v = 8, k = 3$	$v = 7, b = 35, r = 15, k = 3, \lambda = 5$
8	$v = 8, k = 4$	$v = 7, b = 35, r = 20, k = 4, \lambda = 10$
9	$v = 8, k = 5$	$v = 7, b = 21, r = 15, k = 5, \lambda = 10$
10	$v = 8, k = 6$	$v = 7, b = 7, r = 6, k = 6, \lambda = 5$
11	$v = 9, k = 3$	$v = 8, b = 56, r = 21, k = 3, \lambda = 6$
12	$v = 9, k = 4$	$v = 8, b = 70, r = 35, k = 4, \lambda = 15$
13	$v = 9, k = 5$	$v = 8, b = 56, r = 35, k = 5, \lambda = 20$
14	$v = 9, k = 6$	$v = 8, b = 28, r = 21, k = 6, \lambda = 15$
15	$v = 9, k = 7$	$v = 8, b = 8, r = 7, k = 7, \lambda = 6$
16	$v = 10, k = 3$	$v = 9, b = 84, r = 28, k = 3, \lambda = 7$
17	$v = 10, k = 4$	$v = 9, b = 126, r = 56, k = 4, \lambda = 21$
18	$v = 10, k = 5$	$v = 9, b = 126, r = 70, k = 5, \lambda = 35$
19	$v = 10, k = 6$	$v = 9, b = 84, r = 56, k = 6, \lambda = 35$
20	$v = 10, k = 7$	$v = 9, b = 36, r = 28, k = 7, \lambda = 21$
21	$v = 10, k = 8$	$v = 9, b = 9, r = 8, k = 8, \lambda = 7$
22	$v = 11, k = 8$	$v = 10, b = 45, r = 36, k = 8, \lambda = 28$
23	$v = 11, k = 9$	$v = 10, b = 10, r = 9, k = 9, \lambda = 8$
24	$v = 12, k = 9$	$v = 11, b = 55, r = 45, k = 9, \lambda = 36$
25	$v = 12, k = 10$	$v = 11, b = 11, r = 10, k = 10, \lambda = 9$

3. Conclusion

In conclusion, the study was able to establish that for un-reduced BIBD with parameter v and $k \geq 3$. When all the blocks from the design containing treatment i are deleted from the design and the remaining blocks without the treatment i are left, then, the remaining blocks form a BIBD with parameters $v^* = v - 1, b^* = \frac{(v-1)!(v-k)}{k!(v-k)!}, k^* = k, r^* = \frac{(v-2)!(v-k)}{(k-1)!(v-k)!}, \lambda^* = \frac{(v-3)!(v-k)}{(k-2)!(v-k)!}$. This class of BIBD is known as Residual Reduced Balanced Incomplete Block Design. The study was therefore able to derive a new technique for constructing BIBDs from existing BIBDs which adds to the list of techniques that could be used in constructing BIBDs.

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