MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS

2021/ 2022 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES.

DEGREE IN APPLIED STATISTICS WITH COMPUTING.

COURSE CODE: STA 4136

COURSE TITLE: MEASURE AND PROBABILITY THEORY

DATE: **TIME:**

INSTRUCTIONS TO CANDIDATES

Answer Question ONE and any other TWO questions

This paper consists of FOUR printed pages. Please turn over

Question One (30 marks)

(a) Let Ω ={1,2,3} , Ƒ$_1$ = {{1},{2,3},Ω,Ø} and Ƒ$_2$ = {{1,2},{3},Ø,Ω}
Show that Ƒ$_1$ and Ƒ$_2$ are both algebras but not Ƒ$_1$∪ Ƒ$_2$

(5 marks)

(b) Let Ω be a non-empty set. Show that Ƒ = {Ω,Ø} and Ǵ = Ƥ(Ω) = {A: AcΩ} are both σ-algebras

(5 marks)

(c) Define the following terms

(i) A measurable space
(ii) Random variable
(iii) Simple function

(6marks)

(d) Let □ be a measure on algebra Ƒ and let A,B ϵ Ƒ . Then show that (A) ≤ (B) if AcB

(5 marks)

(e) Let Ω be a non-empty set and Ƒ = Ƥ(Ω) . Define (A) = │A│. Show that □ is a measure.

(4 marks)

(f) Prove that Var(X$_1$ + X$_2$ +.... X$_n$) = $\sum_{1=1}^{n} Var(X_i)$ $if\ x_i's$ are independent.

(5 marks)

(g) Let {A$_n$} be a sequence of independent sets. Prove that
P($\bigcap_{i=1}^{\infty} A_i) = \prod_{i=1}^{\infty} P(A_i)$

(5 marks)

Question Two (20 marks)

(a) Let Ω be a non-empty set and {A$_i$}$_{i=N}$ be a sequence of subsets of Ω such that A_{i+1}CA_i for all i ϵ N. Varify that Λ = { A$_i$: I ϵ N } is a π- system.

(8 marks)

(b) Let Ω ={a,b,c,d} and Ƒ$_1$ ={{a},{b,c,d},Ω,Ø} and Ƒ$_2$= { the set of all subsets of Ω}
Define T$_i$: Ω→Ω, I = 1,2
By T$_1$ (ω) = a for ω ϵ Ω
And T$_2$ (ω) =$\begin{cases} a & if\ \ \omega = a, b \\ b & if\ \omega = c, d \end{cases}$
Show that T$_1$ is ⟨Ƒ$_1$,Ƒ$_2$⟩ measurable
And T$_2$ is not ⟨Ƒ$_1$,Ƒ$_2$⟩ measurable (12 marks)

Question Three (20 marks).

(a) Prove that as n increases , the probability that the average of number of successes distributed as Bernoulli deviates from ½ by more than any pre assigned number tends to zero.

(10 marks)

(b) Prove that if X_I are identically independently distributed with $E(X_i) =$ ▯ $< \infty$ then

$$\frac{\sum x_i}{n} \to \mu \text{ as } n \to \infty$$

(10 marks)

Question Four (20 marks)

(a) State and Prove Chebychev's inequality

(10 marks)

(b) Let f be measurable function. Prove that

$$\left| \int f du \right| \leq \int |f| du$$

(10 marks)