# MAASAI MARA UNIVERSITY 

## REGULAR UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR
FOURTH YEAR FIRST SEMESTER

## SCHOOL OF PURE APPLIED AND HEALTH

## SCIENCES

## BACHELOR OF SCIENCE IN CHEMISTRY

## COURSE CODE: CHE 4144

## COURSE TITLE: STATISTICAL THERMODYNAMICS

DATE: $1^{\text {ST }}$ APRIL, 2022 TIME: 1430-1630

## INSTRUCTIONS TO CANDIDATES

1. Answer Question ONE and any other TWO questions.
2. All Examination Rules Apply.
3. Avogardro's costant $=6.022 \times 10^{23} \mathrm{~mol}^{-1}$
4. Gas constant $\mathrm{R}=8.314 \mathrm{JK}^{-1} \mathrm{~mol}^{-1}=0.0821 \mathrm{~L} \mathrm{~atm} \mathrm{~K}^{-1} \mathrm{~mol}^{-1}$
5. Plank constant, $\mathrm{h},=6.626 \times 10^{-34} \mathrm{Js}$
6. Speed of light, $c,=c=2.99 \times 10^{8} \mathrm{Ms}^{-1}$
7. Mass of an electron $=9.11 \times 10^{-31} \mathrm{Kg}$
8. Mass of a proton $=1.7 \times 10^{-27} \mathrm{Kg}$
9. Boltzmann constant $=1.38 \times 10^{-23} \mathrm{JK}^{-1}$
10. $\pi=22 / 7$
$11.1 \mathrm{u}=1.6605 \times 10^{-27} \mathrm{Kg}$
$12 . \mathrm{P}^{\theta}=10^{5} \mathrm{~Pa}$
11. Speed of light, $\mathrm{c},=3.0 \times 10^{8} \mathrm{Ms}^{-1}$

## SECTION A

## Question One (30mks)

a) What is the temperature of a two-level system of energy separation equivalent to $300 \mathrm{~cm}^{-1}$ when the population of the upper state is onehalf that of the lower state (3mks)
b) Derive the expression for the partition function of a two-state system with one state at zero and another state at energy E, given that $\mathrm{q}=\Sigma \mathrm{e}^{-}$ $\beta \mathrm{E}_{\mathrm{i}}$ (3mks)
c) Explain the following terms;
i) Optical trapping (2mks)
ii) Adiabatic demagnetization (2mks)
d) Calculate the translational partition function of a hydrogen molecule confined to a $100 \mathrm{~cm}^{3}$ vessel at $25^{\circ} \mathrm{C}$. Mass of Hydrogen $=2.016 \mathrm{u}$. (4mks)
e) $\quad$ From $E=\sum_{i} n_{i} E_{i}$, show that $E=-N d q / q d \beta(4 m k s)$
f) Calculate the translational partition function at 300 K of a molecule of molar mass $120 \mathrm{~g} \mathrm{~mol}^{-1}$ in a container of volume $2.00 \mathrm{~cm}^{3}$ ( 3 mks )
g) Calculate (a) the thermal wavelength, (b) the translational partition function of an Argon atom in a cubic box of side 1.00 cm at 300 K . Mass of argon = 39.948 u . (5mks)
h) The mean energy for any mode of motion is given by $\left\langle\epsilon^{m}\right\rangle=$ $-\frac{1}{q^{m}}\left(\frac{\partial q^{m}}{\partial \beta}\right)_{V}$, If the translational partition function is given by; $q^{T}=\frac{X}{\Lambda}$ where $\Lambda=\mathrm{h}\left(\frac{1}{2 \pi m}\right)^{\frac{1}{2}}(\beta)^{\frac{1}{2}}$. Show that $\left\langle\epsilon^{m}\right\rangle=\frac{1}{2} k T$ (4mks)

## SECTION B

## Answer any TWO questions from this section, each question carries 20 marks

## Question Two (20mks)

a) Define a canonical ensemble (1 $\mathbf{m k}$ )
b) Evaluate the rotational partition function of $\mathrm{H}-\mathrm{Cl}$ at $25^{\circ} \mathrm{C}$ given that B $=10.59 \mathrm{~cm}^{-1}$, KT/hC $=207.22 \mathrm{~cm}^{-1}$ ( 3 mks )
c) Calculate the standard molar entropy of Neon gas at 200 K Mass of Neon $=20.1797$ u (5mks)
d) Define the following terms as used in statistical thermodynamics;
i) A configuration ( $\mathbf{1} \mathbf{m k}$ )
ii) The weight of a configuration (1mk)
e) Use the equipartition theorem to estimate the constant-volume molar heat capacity of $\mathrm{O}_{3}$ (2mks)
f) Calculate the rotational partition function of $\mathrm{SO}_{2}$ at 298 K from its rotational constants $2.02736 \mathrm{~cm}^{-1}, 0.34417 \mathrm{~cm}^{-1}$, and 0.293535 $\mathrm{cm}^{-1} . \sigma=2$ (3mks)
g) Estimate the molar constant-volume heat capacity of water vapour at $100{ }^{\circ} \mathrm{C}$. Vibrational wave numbers are $3656.7 \mathrm{~cm}^{-1}, 1594 \mathrm{~cm}^{-1}$ and $3755.8 \mathrm{~cm}^{-1}$. The rotational constants of a water molecule are 27.9 , 14.5 and $9.3 \mathrm{~cm}^{-1}$. (4mks)

## Question Three (20mks)

a) Calculate the rotational partition function of $\mathrm{H}_{2} \mathrm{O}$ at 298 K from its rotational constants $27.878 \mathrm{~cm}^{-1}, 14.509 \mathrm{~cm}^{-1}$, and $9.287 \mathrm{~cm}^{-1} . \sigma=2$ (3mks)
b) Calculate the value of $\mathrm{G}_{\mathrm{m}}^{\theta}-\mathrm{G}_{\mathrm{m}}^{\theta}(0)$ for $\mathrm{H}_{2} \mathrm{O}_{(\mathrm{g})}$ at 1500 K given that, $\mathrm{q}^{\mathrm{v}}=$ $1.352, q^{R}=486.7$ and mass of water $=18.015 u$. Density of water $=$ $1 \mathrm{gcm}^{-3}$. (7mks)
c) Evaluate the equilibrium constant for the dissociation $N a_{2(g)}^{\leftrightarrows} \leftrightarrows 2 N a_{(g)}$ at 1000 K from the following data. $\mathrm{D}_{0}=70.4 \mathrm{Kjmol}^{-1}$.

The sodium atoms have doublet ground terms. $\mathrm{P}^{\theta}=1.0 \times 10^{5} \mathrm{~Pa}$
(4mks)

$$
\begin{array}{ll}
\Lambda\left(\mathrm{Na}_{2}\right)=8.14 \mathrm{pm} & \Lambda(\mathrm{Na})=11.5 \mathrm{pm} \\
\mathrm{q}^{\mathrm{R}}\left(\mathrm{Na}_{2}\right)=2246 & \mathrm{q}^{\mathrm{V}}(\mathrm{Na})=4.885 \\
\mathrm{~g}\left(\mathrm{Na}_{2}\right)=1 & \mathrm{~g}(\mathrm{Na})=2
\end{array}
$$

d) Given that $3 / 2 \mathrm{nRT}=3 \mathrm{~N} / 2 \beta$, show that $\beta=1 / \mathrm{KT}$ (3mks)
e) Calculate the proportion of $\mathrm{I}_{2}$ molecules in their second and third vibrational states at $25^{\circ} \mathrm{C}$. The vibration wave number is $214.6 \mathrm{~cm}^{-1}$ (3mks)

## Question Four (20mks)

a) Calculate the standard molar entropy of xenon gas at 100 K . Mass of Xenon = 131.293 u (2mks)
b) Calculate the standard molar entropy of gaseous Argon at $25{ }^{\circ} \mathrm{C}$. Let $\mathrm{P}^{\ominus}=1.0 \times 10{ }^{5} \mathrm{~Pa}$. Mass of Argon $=39.948 \mathrm{u}$ (3mks)
c) Given that $p=N K T / q(d q / d V)$, show that $P=n R T / V(3 m k s)$
d) Determine the expression for proportions of molecules in the ground state, first excited state for a two-level system given that the partition function for a two-level system is given by $q=1+e^{-\beta E}$ ( $3 \mathbf{m k s}$ )
e) The wave numbers of three modes of $\mathrm{CO}_{2}$ modes of vibration are $1388 \mathrm{~cm}^{-1}, 667 \mathrm{~cm}^{-1}$, and $2349 \mathrm{~cm}^{-1}$. Calculate the Vibrational partition function given that $\mathrm{kT} / \mathrm{hC}=1050.42 \mathrm{~cm}^{-1}$. ( 4 mks )
f) Explain the origin of residual entropy (2mks)
g) Estimate the rotational partition function of $\mathrm{O}_{2}$ at $25^{\circ} \mathrm{C}$, given that $\mathrm{B}=$ . (3mks)

