MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2018/2019 ACADEMIC YEAR
SECOND YEAR TWO SEMESTER

SCHOOL OF SCIENCE
Bsc. MATHEMATICS

COURSE CODE: MAT 2215
COURSE TITLE: GROUP THEORY 1

DATE: -4-2019 **TIME: 8:30-10:30PM**

INSTRUCTIONS TO CANDIDATES

1. Answer Question **ONE** and any other **TWO** questions.
2. All Examination Rules Apply.

MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2018/2019 ACADEMIC YEAR
THIRD YEAR TWO SEMESTER

SCHOOL OF SCIENCE
Bsc. MATHEMATICS

COURSE CODE: MAT 411
COURSE TITLE: FIELD THEORY

DATE: -4-2019 **TIME:**

INSTRUCTIONS TO CANDIDATES

1. Answer Question **ONE** and any other **TWO** questions.
2. All Examination Rules Apply.

MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2018/2019 ACADEMIC YEAR
THIRD YEAR TWO SEMESTER

SCHOOL OF SCIENCE
Bsc. MATHEMATICS

COURSE CODE: MAT 3228
COURSE TITLE: RINGS AND MODULES

DATE: 17TH APRIL 2019 **TIME: 1100 – 1300HRS**

INSTRUCTIONS TO CANDIDATES

1. Answer Question **ONE** and any other **TWO** questions.
2. All Examination Rules Apply.

MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2018/2019 ACADEMIC YEAR
THIRD YEAR TWO SEMESTER

SCHOOL OF SCIENCE
Bsc. MATHEMATICS

COURSE CODE: MAT 3228
COURSE TITLE: RINGS AND MODULES

DATE: -4-2019 **TIME: PM**

INSTRUCTIONS TO CANDIDATES

1. Answer Question **ONE** and any other **TWO** questions.
2. All Examination Rules Apply.

Question 1 [30 marks]

1 (a). Give:

i. The definition of a ring.

ii. an example of a finite non-commutative ring with identity.
iii. an example of an infinite non commutative ring. [7 marks]

1(b) i. Let R be a ring. State what is meant by a right $R -$ module.

ii. If R is the ring of 3 by 3 matrices over $\mathbb{Z}_2$. Determine the number of elements in R and suggest a possibility for a left and right $R -$ module. Support your suggestion. [7 marks]

1 (c) i. Give the definition of an integral domain.

ii. Give an example of an integral domain.

iii. Show that $\mathbb{Z}_m[x]$ (polynomials in x over $\mathbb{Z}_m$) is in general not an integral domain. [6 marks]

1 (d). Give the definition of a Euclidean ring and give an example of a ring that satisfies your definition. [4 marks]

1 (e). Let $\Phi: R \to S$ be a ring homomorphism and

$K = \{r \in R/\phi(r) = 0\}$

Prove that K is an ideal of R. [2 marks]

1(f). Test for irreducibility. Factorize if not irreducible.

i. $x^3 + x^2 + 1$ in $\mathbb{Z}_2[x]$
ii. $x^4 + x^2 + 1$ in $\mathbb{Z}_3[x]$ [4 marks]

<u>Question 2</u> [20 marks]

2 a. Prove that every ideal in a Euclidean ring is principal.

b. Find a single generator for the ideal :

i. $204\mathbb{Z} + 78\mathbb{Z} + 364\mathbb{Z}$ of $\mathbb{Z}$.
ii. $(2x^2 + 3x - 5)\mathbb{Q}[x] + (4x^2 + 8x - 5)\mathbb{Q}[x]$ of $\mathbb{Q}[x]$.

<u>Question 3</u> [20 marks]

Let be the ring of 3×3 matrices over $\mathbb{Z}$ and S be the ring of 2×2 matrices over $\mathbb{Z}$.

i. Prove that the additive abelian group M of all 2 by 3 matrices over $\mathbb{Z}$ is a left $S -$ module and a right $R -$ module.
ii. Determine the result of

left action by $\begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$ and

right action by $\begin{bmatrix} 2 & 1 & -1 \\ 3 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix}$

on the module element $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix}$.

iii. Determine a left $S -$submodule of M which is not a right $R -$ submodule.
iv. Determine also a proper subgroup of M which is both a left $S -$submodule and a right $R -$ submodule.

<u>Question 4</u> [20 marks]

a) Prove that every principal ideal domain satisfies the ascending chain condition on ideals.
b) Prove that every principal ideal domain is a unique factorization domain.
c) Demonstrate that any ascending chain of ideals of $\mathbb{Z}$ which contain the ideal $9000\mathbb{Z}$ must terminate in a finite number of steps and state the maximum number of possible steps.

Question 5 [20 marks]

5 (a). Give the definition of a ring homomorphism Φ and state what is meant by:

i. the kernel of Φ.
ii. the image of Φ.

5 (b). If $\Phi: R \to S$ is a ring homomorphism prove that :

i. the kernel of Φ is an ideal of R.
ii. the image of R under Φ is a subring of S.
iii. the ring $\frac{R}{\ker \Phi}$ is isomorphic to the image of R.

5 (C). Illustrate all parts of 5(b) by a ring homomorphism of your choice.