MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2018/2019 ACADEMIC YEAR
FOURTH YEAR SECOND SEMESTER

SCHOOL OF SCIENCE
BACHELOR OF SCIENCE

COURSE CODE: MAT 414
COURSE TITLE: TOPOLOGY II

DATE: 18-4-2019 **TIME: 11:00-13:00HRS**

INSTRUCTIONS TO CANDIDATES

Answer Question **ONE** and any other **TWO** questions

*This paper consists of **TWO** printed pages. Please turn over.*

QUESTION ONE – 30 MARKS

a) Define a T_1-space, hence deduce whether the topological space (X, τ) where $\tau = \{X, \varnothing, \{a, b\}, \{b\}, \{b, c\}\}$ is a topology defined on $X = \{a, b, c\}$ is a T_1-space. **(4 marks)**

b) Prove that every T_4-space is a Tychonoff space. **(6 marks)**

c) Define first countability property, hence show that every metric space satisfies first countability axiom. **(5 marks)**

d) Show that any finite subset of a topological space (X, τ) is compact. **(5 marks)**

e) Show that connectedness is a topological property. **(5 marks)**

f) Define a homotopy between continuous functions *f* and *g* defined on R.
Hence, show that if $f, g : \mathrm{R} \to \mathrm{R}$ are any two continuous real functions and $F : \mathrm{R} \times [0,1] \to \mathrm{R}$ is a function defined by $F(x,t) = (1-t) \cdot f(x) + t \cdot g(x)$, then *F* is a homotopy between *f* and *g*. **(5 marks)**

QUESTION TWO – 20 MARKS

a) Prove that every subspace of a second countable space is second countable. **(3 marks)**

b) Prove that the class $C(X, \mathrm{R})$ of all real-valued continuous functions on a completely regular T_1-space separates points. **(5 marks)**

c) Define a separable space, hence show that the discrete space (X, τ) is separable if and only if X is countable. **(4 marks)**

d) Prove that a topological space (X, τ) is a T_1-space if and only if every singleton set of X is closed. **(8 marks)**

QUESTION THREE – 20 MARKS

a) Show that any compact subset of a T_2-space is closed. **(3 marks)**

b) Show that if the function *f* is homotopic to *g* ($f \simeq g$), then *g* is also homotopic to *f* ($g \simeq f$). **(5 marks)**

c) Prove that a continuous image of a path connected set is path connected. **(5 marks)**

d) Prove that regularity is a hereditary property. **(7 marks)**

QUESTION FOUR – 20 MARKS

a) Prove that the union of finite compact subsets of a topological space is also compact. **(5 marks)**

b) Differentiate between a T_3-space and T_4-space, hence show that every T_4-space is a T_3-space. **(8 marks)**

c) Show that first countability property is a topological property. **(7 marks)**

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