# MAASAI MARA UNIVERSITY

## REGULAR UNIVERSITY EXAMINATIONS
## 2018/2019 ACADEMIC YEAR
## SECOND YEAR TWO SEMESTER

## SCHOOL OF SCIENCE
## BSC. MATHEMATICS

## COURSE CODE: MAT 2215
## COURSE TITLE: GROUP THEORY 1

**DATE: 25TH APRIL, 2019** **TIME: 0830 – 1030 HRS**

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**INSTRUCTIONS TO CANDIDATES**

1. Answer Question **ONE** and any other **TWO** questions.
2. All Examination Rules Apply.

**QUESTION 1 [30 MARKS]**

1 a). Give an example of:

i. an associative binary operation on a set.
ii. anon-associative binary operation.

In each case specify the operation and the set, and support your claim.

[5 Marks]

1 b). (i.) Give the meaning of the statement:

"$G$ is a non –commutative group with two generators".

i. Show that $S_3$ a non-commutative group.
ii. Give an example of a group of order 4 with two generators.

[6 Marks]

1 c). State the ring axioms and give an example of a ring with a finite number of elements. [5 Marks]

1 d). i. Write down the elements of the field $\mathbb{Z}_5$ and construct a multiplication table for the field.

ii. Solve the equation $x^2$=4 over $\mathbb{Z}_5$. [6Marks]

1 e). Give the definition of:

i. Subgroup.
ii. Coset.
iii. Factor group.

Illustrate using a group of your choice. [8 Marks]

## QUESTION 2 [20 MARKS]

Let $A=\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ , $B=\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ be matrices over $\mathbb{Q}$:

i. Determine $A^2$ , $B^2$ ,$AB$ ,$BA$ , $(AB)^{15}$ , $(AB)^n$.

ii. List all elements of multiplicative group $V$ generated by $A$ and .

iii. If $\begin{bmatrix} 0 & b \\ c & 0 \end{bmatrix}^2 = B$ , determine possible integer values of $b$ and $c$ and the order of the group generated by $A$ and $\begin{bmatrix} 0 & b \\ c & o \end{bmatrix}$ .

## QUESTION 3 [20 MARKS]

3 a. State and prove Lagrange an finite groups.

3 b. Let $D = \langle a, b \mid a^2 = b^3 = e, ba = ab^2 \rangle$

i. List the elements of $D$ .

ii. List all the subgroups of $D$ .

iii. Choose a subgroup of order 2, of $D$ and use it to illustrate Lagrange`s theorem.

## QUESTION 4 [20 MARKS]

4 a. Let $R$ be the ring $\langle \mathbb{Z}_{12}; +_{12,} \times_{12} \rangle$:

i. Write down all the non-zero divisors of zero in $R$ .

ii. Determine all the elements with multiplicative inverses and show that they form a group.

iii. Determine the ideals of $R$ .

iv. For each ideal in iii. Determine the corresponding factor ring.

4 b. Let $C$ be the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ in the ring $R$ of $2 \times 2$ matrices

Over $\mathbb{Z}$;

i. Write down 5 matrices $D$ such that $CD = 0$ and $D \neq 0$.
ii. Determine whether the set of all such matrices $D$ such that $CD = 0$ is a right ideal of the ring $R$ .

**QUESTION 5 [20 MARKS]**

5 a. Give an example of:

i. A finite field with $n$ elements where $5 < n < 9$ .
ii. A field with an infinite number of elements.
iii. A field with an uncountable number of elements.

5 b. Let $\{f(x) = x^2 + x + 1\}$ be a polynomial in$\mathbb{Z}_2$[x], and

let α be a root of f(x).

i. Show that {0, 1, α, α+1} is a field with 4 elements.
ii. Solve the equation $x^3 - 1 = 0$ in the field.

5 c. A function:

F: $\mathbb{Z}_{12} \rightarrow \mathbb{Z}_{12}$ is given by $f(x) = 4x$.

Determine the image of $f$ and the set of elements that are mapped to zero.

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