

# MAASAI MARA UNIVERSITY 

UNIVERSITY EXAMINATIONS 2018/2019 (REGULAR)

SCHOOL OF SCIENCE AND INFORMATION SCIENCES

# UNIVERSITY EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE (COMPUTER SCIENCE) 

SECOND YEAR FIRST SEMESTER EXAMINATION

COURSE CODE: COM 1205
COURSE TITLE: DISCRETE STRUCTURE II

## DATE: 17TH APRIL 2019

TIME: 11.00AM TO 01.00PM INSTRUCTIONS

Answer Questions ONE and any other TWO

## SECTION - A QUESTION ONE (COMPULSORY 30 MARKS)

a) Convert the following SOP expression to an equivalent POS expression.
$\mathrm{ABC}+\mathrm{ABC}+\mathrm{ABC}+\mathrm{ABC}+A B \mathrm{C}$
(2 Marks)
b) Construct logic networks for the following Boolean expressions, using AND gates, OR gates, and inverters. $(x+y) z$
(2 Marks)
c) A group consists of nine men and six women. Find the number m of committees of six that can be selected from the class.
(3 Marks)
d) Verify that the proposition $p \vee(p \wedge q)$ is not tautology.
(4 Marks)
e) Use the K-Map and convert the expression into minimal form.
$A B C D+A B C D+A B C D+A B C D+\mathrm{B} C D+\mathrm{BCD}+\mathrm{ABCD}+\mathrm{ABD}+\mathrm{ABCD}$
(4 Marks)
f) Determine the values of $\mathrm{A}, \mathrm{B}, \mathrm{C}$, and D that make the sum term $A+\mathrm{B}+C+\mathrm{D}$ equal to zero.
(4 Marks)
g) Which of the following expressions is in the sum-of-products (SOP) form?

1. $(A+B)(C+D)$
2. (A)B(CD)
3. $\mathrm{AB}(\mathrm{CD})$
4. $A B+C D$
(2 Marks)
h) Derive the Boolean expression for the logic circuit shown below:

(3 Marks)
i) Compute the truth table of $(F \vee G) \wedge \neg(F \wedge G)$.
(6 Marks)
a) Prove $\mathrm{x}+\bar{y}=\mathrm{x}+(\overline{=} \bar{x} \cdot \bar{y}+\bar{x} \cdot \bar{y})$
(5 Marks)
b) Let's consider a propositional language where

- p means "Paola is happy",
- q means "Paola paints a picture",
- r means "Renzo is happy".

Formalize the following sentences:

1. "if Paola is happy and paints a picture then Renzo isn't happy"
2. "if Paola is happy, then she paints a picture"
3. "Paola is happy only if she paints a picture"
(3 Marks)
c) From the truth table below, determine the standard SOP expression.

| Inputs |  |  | Output |
| :---: | :---: | :---: | :---: |
| A | B | C | X |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

(4 Marks)
d) Use the truth tables method to determine whether $(p \rightarrow q) \vee(p \rightarrow \neg q)$ is valid.
(4 Marks)
e) Let's consider a propositional language where

- p : means " x is a prime number",
- q: means "x is odd". Formalize the following sentences:

1. " $x$ being prime is a sufficient condition for $x$ being odd"
2. " $x$ being odd is a necessary condition for $x$ being prime"
(4 Marks) QUESTION THREE (20 MARKS)
a) Prove the associative law: $(\mathrm{p} \wedge \mathrm{q}) \wedge \mathrm{r} \equiv \mathrm{p} \wedge(\mathrm{q} \wedge \mathrm{r})$
(3 Marks)
b) Design a three-input minimal AND-OR circuit L that will have the following truth table:

$$
T=[A=00001111, B=00110011, C=01010101 ; L=11001101]
$$

(5 Marks)
c) Reduce to Negative Normal Form (NNF) the formula.

$$
\begin{equation*}
\neg(\neg \mathrm{p} \vee q) \vee(\mathrm{r} \rightarrow \neg \mathrm{~s}) \tag{2Marks}
\end{equation*}
$$

d)Applying DE Morgan's theorem to the expression $A B C$, we get $\qquad$ .
(2 Marks)
e) A truth table for the SOP expression $\mathrm{ABC}+\mathrm{ABC}+A B \mathrm{C}$ has how many input combinations?
(2 Marks)
f) Use the K-Map and convert the expression into minimal form.
$A B C D+A B C D+A B C D+A B C D+\mathrm{ABCD}+\mathrm{ABCD}+A B C D+\mathrm{ABCD}$
(6 Marks)

## QUESTION FOUR (20 MARKS)

a) Simplify the expression in to minimal form.

$$
\begin{equation*}
\mathrm{Z}=\mathrm{f}(\mathrm{~A}, \mathrm{~B}, \mathrm{C})=\overline{\mathrm{A}} \overline{\mathrm{~B}} \overline{\mathrm{C}}+\overline{\mathrm{A}} \mathrm{~B}+\mathrm{AB} \overline{\mathrm{C}}+\mathrm{AC} \tag{2Marks}
\end{equation*}
$$

b) Use the truth tables method to determine whether the formula $\varphi: p \wedge \neg q \rightarrow p \wedge q$ is a logical consequence of the formula $\psi: \neg \mathrm{p}$.
c) Draw a logic circuit for $\mathrm{AB}+\mathrm{AC}$.
d) Define an appropriate language and formalize the following sentences using FOL formulas.

1. All Students are smart.
2. There exists a student.
3. There exists a smart student.
4. Every student loves some student.
5. Every student loves some other student.
6. There is a student who is loved by every other student.
7. Bill is a student.
8. Bill takes either Analysis or Geometry (but not both).
9. Bill takes Analysis and Geometry.
10. No students love Bill.
e) Use the truth tables method to determine whether $p \rightarrow(q \wedge \neg q)$ and $\neg p$ are logically equivalent.
f) Define a propositional language which allows to describe the state of a traffic light on different instants.
With the language defined above provide a (set of) formulas which expresses the following facts:
11. the traffic light is either green, or red or orange;
12. the traffic light switches from green to orange, from orange to red, and from red to green;
13. it can keep the same color over at most 3 successive states.
