MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2018/2019 ACADEMIC YEAR
SECOND YEAR SECOND SEMESTER

SCHOOL OF SCIENCE
BACHELOR OF SCIENCE IN APPLIED STATISTICS WITH COMPUTING

COURSE CODE: STA 2216
COURSE TITLE: FINANCIAL MATHEMATICS I

DATE: 15TH APRIL 2019
TIME: 1100 - 1300 HOURS
INSTRUCTIONS TO CANDIDATES

1. Answer Question **ONE** and any other **TWO** questions.
2. Show all your Workings.

*This paper consists of 4 printed pages. Please turn over.*

QUESTION 1

a) Define the term financial management and state the three decision functions that are vested with financial manager. **[4 Marks]**

b) Differentiate between effective interest rate and Nominal interest rate. **[2 Marks]**

c) Find the value at interest rate of 5 % per annum effective for the following functions; **[12 Marks]**

(i) \( a_{65} \) (ii) \( s_{62} \) (iii) \( a_{63} \) (iv) \( \bar{a}_{21} \)

d) John Grisham is considering investing in a security that has the following tribulations of possible one year returns:

<table>
<thead>
<tr>
<th>Probability of occurrence</th>
<th>0.10</th>
<th>0.2</th>
<th>0.3</th>
<th>0.3</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Possible returns</td>
<td>-0.10</td>
<td>0.0</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
</tr>
</tbody>
</table>

What is the expected return and the standard deviation associated with the investment **[6 Marks]**

e) Suppose that the force of interest per annum at time \( t \) years is

\[
\delta(t) = a e^{-bt}
\]

Show that the present value of 1 due at time \( t \) years is
\[ v(t) = \exp \left( \frac{a}{b} (e^{-bt} - 1) \right) \]

**Marks**

**f)** Differentiate between the terms **Annuity** and **Perpetuity** as used in financial mathematics.  
**[2 Marks]**
QUESTION 2  

a) Assume that \( \delta(t) \), the force of interest per annum at time \( t \) (years), is given by the formula

\[
\delta(t) = \begin{cases} 
0.08 & 0 \leq t < 5 \\
0.06 & 5 \leq t < 10 \\
0.04 & t \geq 10 
\end{cases}
\]

Derive expressions for \( v(t) \), the present value of 1 due at time \( t \).

b) An investor effects’ a contract under which he will pay 15 premiums annually in advance into an account which accumulates according to the above force of interest. Each premium will be of amount £900 and the first premium will be paid at time 0. In return the investor will receive either

(i) The accumulated amount of the account one year after the final premium is paid: or

(ii) A level annuity payable annually for eight years, the first payment being made one year after the final premium is paid.

(iii) Find the lump sum payment under option (i) and the amount of the annual annuity under option (ii)

[20 Marks]

QUESTION 3  

Two project proposals for electricity installation in an institution were presented to you as a financial advisor of a certain consultant firm;

**Project X:** delegates all installations tasks to a tendered company. The estimated cash flow for project X, are;

<table>
<thead>
<tr>
<th>Time period</th>
<th>Estimated cost</th>
<th>Nature of charges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beginning of year 1</td>
<td>$(150,000)$</td>
<td>Contractors fee</td>
</tr>
</tbody>
</table>
Project Y: proposes that all the installations work is done in-house by purchasing the required implements and use of own staff. The estimated cash-flow for this project are,

<table>
<thead>
<tr>
<th>Time period</th>
<th>Estimated cost</th>
<th>Nature of charges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beginning of year 1</td>
<td>($325,000)</td>
<td>Staff cost</td>
</tr>
<tr>
<td>Throughout year 1</td>
<td>($75,000)</td>
<td>Staff cost</td>
</tr>
<tr>
<td>Throughout year 2</td>
<td>($90,000)</td>
<td>Staff cost</td>
</tr>
<tr>
<td>Throughout year 3</td>
<td>($120,000)</td>
<td>Staff cost</td>
</tr>
<tr>
<td>End of year 3</td>
<td>$1,000,000</td>
<td>Sales</td>
</tr>
</tbody>
</table>

Values in brackets indicates expenses or cash out flows, whereas in project Y the cost throughout the year are assumed to be spread evenly within the year.

Required: Discriminate between the two projects using;

(i). Net present Value, and

(ii). Internal rate of return

QUESTION 4

a) If $\delta(t)$ and $A(t_o, t)$ are continuous functions of $t$ for $t_o \leq t$, and the principle of consistency holds for $t_o \leq t_1 \leq t_2$. Proof that, $A(t_1, t_2) = \exp \int_{t_1}^{t_2} \delta(t) dt$. [7 Marks]

b) Given that

$$\ddot{a}_{\overline{n}} = 7.029584 \quad \text{and} \quad \ddot{a}_{2\overline{n}} = 10.934563$$
find the rate of interest $i$ and duration $n$. [6 Marks]

c) A bank lends a company £ 5,000 at a fixed rate of interest of 10 % per annum. The loan is to be repaid by five level annual payments. Calculate the interest and capital payments at each repayment date. [7 Marks]