# MAASAI MARA UNIVERSITY

## REGULAR UNIVERSITY EXAMINATIONS
## 2018/2019 ACADEMIC YEAR
## SECOND YEAR SECOND SEMESTER

## SCHOOL OF SCIENCE
## BACHELOR OF SCIENCE IN APPLIED STATISTICS WITH COMPUTING

## COURSE CODE: STA 2112
## COURSE TITLE: MATHEMATICAL STATISTICS I

**DATE: 16TH APRIL 2019** **TIME: 1100 - 1300 HRS**

## INSTRUCTIONS TO CANDIDATES

1. Answer Question **ONE** and any other **TWO** questions.
2. Show all your Workings.

***This paper consists of 5 printed pages. Please turn over.***

## QUESTION 1

a). Let $X_1$ and $X_2$ be jointly distributed random variables with the density

$$f(x_1, x_2) = \begin{cases} 4e^{-(x_1+3x_2)} & 0 < x_2 < x_1 < \infty \\ 0 & otherwise \end{cases}$$

Find,

**i).** the marginal density of $X_1$ and $X_2$. **[4 Marks]**

**ii).** the mean of $X_1$ and $X_2$. **[3 Marks]**

b). A certain market has both an express check-out line and a super-express checkout line. Let $X_1$ denote the number of customers in line at the express check-out at a particular time of day, and let $X_2$ denote the number of customers in line at the super express checkout at the same time. Suppose the joint probability mass function of $X_1$ and $X_2$ is given in the following table.

| $x_1$ \ $x_2$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0.03 | 0.01 | 0.02 | 0.04 |
| 1 | 0.01 | 0.04 | 0.10 | 0.05 |
| 2 | 0.00 | 0.02 | 0.05 | 0.03 |
| 3 | 0.04 | 0.04 | 0.10 | 0.02 |
| 4 | 0.05 | 0.03 | 0.20 | 0.12 |

Determine the probability that;

i). there is exactly one customer in each line? **[1 Marks]** ii). the number of customers in the two lines are identical? **[1 Marks]**

iii). the total number of customers in the two lines is at least four? **[2 Marks]**

c). Let the random variable $X$ denote the time until a computer server connects to your machine (in milliseconds), and let $Y$ denote the time until the server authorizes you as a valid user (in milliseconds). Each of these random variables measures the wait from a common starting time and $X < Y$. Assume that the joint probability density function for $X$ and $Y$ is

$$f(x,y) = \begin{cases} k\exp(-0.001x - 0.002y) & 0 < x < y < \infty \\ 0 & elsewhere \end{cases}$$

Determine;

**i).** The constant *k*. **[4 Marks]**

**ii).** The probability $X$ is less than 1000 and $Y$ is less than 2000. **[4 Marks]**

**iii).** The probability $Y$ is greater than 2000. **[4 Marks]**

d). nut company markets cans of deluxe mix nuts containing almonds, cashew nuts and peanuts. Spouse that the weight of each can is exactly one kilogram, but the weight contribution of each type of nut is random. Because the three weights sum to one, a joint probability distribution model for any two gives all necessary information about the weight of the third. Let $X$ be the weight of almond and $Y$ the weight of cashews in a randomly selected can. The joint density is given as;

$$f(x,y) = \begin{cases} 24xy & 0 \le x \le 1\ ;\ 0 \le y \le 1;\ x + y \le 1 \\ 0 & elsewhere \end{cases}$$

Determine;

i). The probability that the two types of nuts together make at most 50 %. **[3 Marks]**

ii). The covariance of X and Y. **[4 Marks]**

## QUESTION 2

a). Let $X$ and $Y$ be two random variables whose joint probability density function is a Bivariate normal distribution. If $E(X) = 2E(Y)$ and

$$f(x,y) = \begin{cases} k\ exp - \frac{1}{2}(ax^2 + by^2 - 2cxy), & -\infty < x < \infty : \infty < y < \infty \\ 0, & elsewhere \end{cases}$$

Where *a, b* and *c* are constants. Find;

i). The variance covariance matrix of the variates clearly stating the conditions that must be satisfied by *a, b* and *c* for $f(x,y)$ to exist. **[9 Marks]**

ii). The value of the constant *k*. **[2 Marks]** b). Two continuous random variables X and Y have joint probability density function

$$f(x,y)=\begin{cases}\alpha^2 e^{-\alpha y}, & 0 \le x \le y\\ 0, & otherwise\end{cases}$$

Determine,

**i).** The conditional mean of X given Y. **[5 Marks]**

**ii).** The conditional variance of X given Y. **[4 Marks]**

**QUESTION 3**

a). In an insurance firm the number of contracts entered into in a year (in hundreds) is a random variable $X$ while the profit generated (millions of Kenya Shillings) dependent on these contracts is also a random variable $Y$. If the joint distribution of $X$ and $Y$ is given by,

$$f(x,y)=\begin{cases}\frac{1}{8}(4-2x-y) & 0<x<2\,;\ -5<y<3\\ 0 & elsewhere\end{cases}$$

**i).** Find the profit the profits generated given that there are 100 contracts during the year. **[6 Marks]**

**ii).** Find the marginal distribution of $Y$. **[3 Marks]**

b). Let X and Y be continuous random variables with joint probability density function.

$$f(x,y)=\begin{cases}e^{-x}, & 0<y<x<\infty\\ 0, & elsewhere\end{cases}$$

Determine,

i). $m(t_1, t_2)$ , **[6 Marks]**

ii). $\text{cov}(X, Y)$ **[5Marks]**

## QUESTION 4

a). A rectangle has the edge lengths $X_1$ and $X_2$, where $X_1$ and $X_2$, are independent random variables, both with density

$$f(x_1, x_2) = \begin{cases} 3x^2, & 0 < x < 1 \\ 0, & \textit{otherwise.} \end{cases}$$

Define new random variables $Y_1 = X_1 X_2$ and $Y_2 = \frac{X_1}{X_2}$

Determine the density of $Y_1$ and $Y_2$. **[5 Marks]**

b). First a point $Y$ is selected at random from the interval (0, 1). Then another point $X$ is chosen at random from the interval (0, Y). Find the density of $Y$.

**[5 Marks]**

c). The life times of Vehicle Head lamps manufactured by company A are random variable with the following density function:

$$f(x) = \begin{cases} \frac{1}{6} e^{-x/6}, & x > 0 \\ 0, & \textit{otherwise.} \end{cases}$$

The lifetimes of Vehicle Head lamps manufactured by B are random variable with the following density function:

$$f(y) = \begin{cases} \frac{2}{11} e^{-2y/11}, & y > 0 \\ 0, & \textit{otherwise.} \end{cases}$$

Christine buys two Vehicle Head lamps, one from company A and the other from company B, and installs them on her car at the same time. What is the probability that the Vehicle Head lamp from company B outlasts that of company A?

**[10 Marks]**

**//END**