MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS

2018/2019 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SCHOOL OF SCIENCE
BACHELOR OF SCIENCE

COURSE CODE: PHY 415

COURSE TITLE: STATISTICAL MECHANICS

DATE: 16/04/2019 TIME: 08.30 – 1030AM

INSTRUCTIONS TO CANDIDATES

1. Answer Question **ONE** and any other **TWO** questions

2. Use of sketch diagrams where necessary and brief illustrations are encouraged.
3. Read the instructions on the answer booklet keenly and adhere to them.

*This paper consists of **four** printed pages. Please turn over.*

QUESTION ONE (30 MARKS)

(a) (i) Define statistical ensemble as used in statistical mechanics **(2 Marks)**

(ii) Differentiate between the three classifications of ensembles **(7 Marks)**

(iii) The energy per unit volume of a classical ideal gas is given by

$$\frac{E}{N} = \frac{3}{2}kT$$

Use the equation to develop the principle of equipartition of energy of the classical ideal gas **(4 Marks)**

(b) Maxwell-Boltsmann distribution for an ideal gas given by

$$f(p) = Ce^{\frac{\beta p^2}{2m}}$$

Show that the constant C is given by

$$C = n\left(\frac{1}{2\pi mkT}\right)^{\frac{3}{2}}$$

Where

n is the number of particles in consideration

m is the mass of the particles

k is Boltzmann constant

T is the absolute temperature

(7 Marks)

(c) Give an illustration of how the concept of identical particles is treated in quantum statistics
(6 Marks)

(d) State any two differences between Bosons and Fermions, giving one example of each
(4 Marks)

QUESTION TWO (20 MARKS)

(a) Briefly explain the behavior of a Fermi gas at absolute zero temperature
(4 Marks)

(b) Consider a Fermi gas at low temperature limit (T=0) whose internal energy per particle is given by

$$\frac{U_o}{N} = \frac{2}{3}\varepsilon_F$$

Where ε_F is the Fermi Energy.

(i) Name with reason the ensemble that best describes the Fermi gas at that temperature.
(3 Marks)

(ii) Sketch the relationship between the probability of occupation vs the energy for the Fermi gas around the Fermi level at T = 0 and T >0
(4 Marks)

(iii) Explain the distribution as displayed in the sketch you have provided in (ii) above
(2 Marks)

(c) Discuss how a Fermi gas is applied to model electrons in a solid **(7 Marks)**

QUESTION THREE (20 MARKS)

(a) (i) Describe what is meant by Bose- Einstein condensation **(3 Marks)**

(ii) Derive the representation of the cut off frequency for phonons vibrating in a crystal lattice that is used to arrive at the Deby Temperature **(6 Marks)**

(b) Show that for a Bose gas whose photon number is not conserved, the energy per unit volume of the gas is given by

$$n = K\left(\frac{kT}{\hbar c}\right)^3$$

Where c is the speed of light and K is a term defined by

$$K = \frac{1}{\pi^2}\int_0^\infty \frac{x^2}{e^x - 1}dx$$

(11 Marks)

QUESTION FOUR (20 MARKS)

(a) (i) Define the entropy of a gas in a thermally isolated system stating all the parameters **(2 Marks)**

(ii) Find the entropy S(N,V,E)of an ideal gas of N classical mono-atomic particles with a fixed total energy (E) contained in an l-dimensional box

(8 Marks)

(b) A classical gas in a volume V is composed of N non-interacting and indistinguishable particles. The single particle Hamiltonian is

$H = \frac{p^2}{2m} + \varepsilon$ with *m* the mass of the particle and *p* the absolute value of the momentum.

For each particle, there exist two internal energy levels: a ground state with energy $\varepsilon = 0$ and degeneracy g_1, and an excited state with energy $\varepsilon = \varepsilon_1$ and degeneracy g_2.

(i) Determine the canonical and grand canonical partition function for N particles.
(4 Marks)

(ii) Compute the energy E of the total system as a function of the temperature T.
(6 Marks)

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