MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS

2018/2019 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

SCHOOL OF SCIENCE

BACHELOR OF SCIENCE

COURSE CODE: MAT 2214

COURSE TITLE: NUMERICAL ANALYSIS I

DATE: 11TH DECEMBER, 2018 **TIME: 1100 - 1300 HOURS**

INSTRUCTIONS TO CANDIDATES

*Answer **ALL** questions in **Section A** and **ANY** Other **TWO** questions from **Section B***

DO NOT MAKE ANY WRITING ON THIS QUESTION PAPER

*This paper consists of **THREE** printed pages. **Please turn over.***

SECTION A (30 MARKS)

Question one (30 Marks)

a. Use the intermediate value theorem to show that
$x^5 - 2x^3 + 3x^2 - 1 = 0$ has a solution in the interval $[0,1]$ **(3 Marks)**

b. Determine

i. The second and **(4 Marks)**

ii. The third Taylor polynomial for the function $f(x) = \cos x$ about $x_0 = 0$ and use these polynomial to approximate $\cos(0.01)$ **(4 Marks)**

iii. Use the third Taylor polynomial and its remainder term to approximate $\int_0^{0.1} \cos x \, dx$ **(6 Marks)**

c. To determine the number of iterations necessary to solve $f(x) = x^3 + 4x^2 - 10 = 0$ with accuracy $\varepsilon = 10^{-3}$using $a_1 = 1$ and $b_1 = 2$ requires finding an integer N that satisfies;
$|p_N - p| \leq 2^{-N}(b - a) = 2^{-N} < 10^{-3}$ Hence or otherwise determine the number of iterations required to obtain an approximation accurate to within 10^{-3} **(5 Marks)**

d. Use the bisection method to find solution accurate to within $10^{-2}$for $x^3 - 7x^2 + 14x - 6 = 0$ on $[0,1]$ **(8 Marks)**

SECTION B (40 MARKS)

Question two (20 Marks)

a. Using the Newton – Raphson method, approximate a solution to the equation $\cos x - x = 0$ on $\left[0, \frac{\pi}{2}\right]$ **(10 Marks)**

b. Using the modified Newton – Raphson method, find the solutions accurate to within $10^{-7}$to the problem $f(x) = e^x - x - 1$ for $0 \leq x \leq 1$ [HINT: $p_0 = 1$] **(10 Marks)**

Question three (20 Marks)

a. Compute up through third differences of the discrete function displayed by the y_k column in the table below

k	y_k	Δy_k	$\Delta^2 y_k$	$\Delta^3 y_k$
0	1			
1	8			
2	27			
3	64			
4	125			
5	216			
6	343			
7	512			

(5 Marks)

b. Using finite difference table show that:

i. $\Delta^3 y_0 = y_3 - 3y_2 + 3y_1 - y_0$ **(5 Marks)**

ii. $\Delta^4 y_0 = y_4 - 4y_3 + 6y_2 - 4y_1 + y_0$ **(5 Marks)**

c. Calculate differences through the fifth order in the table below

k	0	1	2	3	4	5	6	7
x_k	0	0	0	1	1	0	0	0

(5 Marks)

Question four (20 Marks)

a. Given the function f at the following values:

x	1.8	2.0	2.2	2.4	2.6
$f(x)$	3.12014	4.42569	6.04241	8.03014	10.46675

approximate $\int_{1.8}^{2.6} f(x)dx$ using:

i. Trapezoidal Rule **(5 Marks)**

ii. Simpson's Rule **(5 Marks)**

b. Using the nodes, $x_0 = 2, x_1 = 2.5$ and $x_2 = 4$. Find the second Lagrange interpolating polynomial for $f(x) = \frac{1}{x}$ **(10 Marks)**

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