MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2017/2018 ACADEMIC YEAR
YEAR 3: SEMESTER 2
SCHOOL OF SCIENCE AND INFORMATION SCIENCES
BACHELOR OF SCIENCE/BACHELOR OF EDUCATION-SCIENCE/ARTS/SPECIAL EDUCATION

COURSE CODE: MATH 311
COURSE TITLE: REAL ANALYSIS 2

DATE: 23/8/2018 TIME: 8.30-10.30 A.M (2 HOURS)

INSTRUCTIONS TO CANDIDATES

1. Answer Question **ONE** and any other **TWO** questions

2. Do not write anything on this question paper

QUESTION ONE (30 MARKS)

(a)(i) State the implicit function theorem. (3mks)

(ii) Given $3x^2 - yz^2 - 4xz - 7 = 0$, use the implicit function theorem to show that near $(-1,1,2)$ we can write $y = f(x,z)$ and find $\frac{\partial y}{\partial x}(-1,2)$. (7mks)

(b) (i) Define a monotonic function. (2mk)

(ii) Prove that $f(x) = x^3 - 3x^2 + 3x + 20$ is increasing on $\mathbb{R}$. (4mks)

(c) Let $f_n(x) = \frac{x^n}{2^n}$ define a sequence of functions. Determine whether the sequence converges and find the range of convergence. (4mks)

(d) (i) Prove that if $f: [a,b] \to \mathbb{R}$ is monotonic, then f is of bounded variation (3mks)

(ii) Suppose that f is a non-decreasing function on $[a,b]$. Show that $V_a^x f = f(x) - f(a) \ \forall x \in [a,b]$. (3mks)

(e) State the inverse function theorem of calculus. (4mks)

QUESTION TWO (20 MKS)

(a) Let $f(x) = x^2$ on $[0,1]$ and $\alpha(x) = 2x + 1$ on $[0,1]$.

(i) Compute $U(P,f,\alpha)$ and $L(P,f,\alpha)$ where $P = \{0 < \frac{1}{3} < \frac{2}{3} < 1\}$ (6mks)

(ii) For all $\in \mathbb{N}$, let $P_n = \{0, \frac{1}{2}, \frac{2}{n}, \ldots 1\}$. Compute $\lim_{n\to\infty} U(P,f,\alpha)$ and $\lim_{n\to\infty} L(P,f,\alpha)$ (6mks)

(iii) From (ii) above, state whether or not $f \in \mathcal{R}(\alpha)$ on $[0,1]$ (2mks)

(b) Evaluate

(i) $\int_0^{\pi} x\, d(sin\alpha)$ (3mks)

(ii) $\int_{-1}^{1} x\, de^{|x|}$ (3mks)

QUESTION THREE(20 MKS)

(a) Suppose $f_n \to f$ uniformly on a set E in a metric space. Let a be a limit of E and suppose that $\lim_{x\to a} f_n(x) = A_n$. Prove that A_n converges and $\lim_{x\to a} f(x) = \lim_{n\to\infty} A_n$. (6mks)

(b) Show that a sequence $\{f_n\}$ of functions defined on a set $E \in \mathbb{R}$ converges uniformly on E iff $\forall\ \epsilon > 0$, there exist a number $N \in \mathbb{N}$ such that

$|f_n(x) - f_m(x)| < \epsilon\ \forall\ n \geq N, m \geq N$ and $x \in E$. (8mks)

(c) Use weierstrass M-test to show that $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ is uniformly convergent (6mks)

QUESTION FOUR(20 MKS)

(a) Let $f: [a,b] \to \mathbb{R}$ be bounded and $\alpha: [a,b] \to \mathbb{R}$ be monotonically increasing function. If P is a partition of $[a,b]$, define

(i) Upper Riemann-stieltjes sum of f with respect to α over $[a,b]$. (2mks)

(ii) Lower Riemann-stieltjes sum of f with respect to α over $[a,b]$. (2mks)

(iii) When is the function f said to be Riemann stieltjes integrable? (2mks)

(b) (i)Prove that if f is monotonic on $[a,b]$ and if α is continuous and monotonic on $[a,b]$, then $f \in \mathcal{R}(\alpha)$. (4mks)

(ii) Let $f: [a,b] \to \mathcal{R}$ be bounded. Prove that $f \in \mathcal{R}(\alpha)$ on $[a,b]$ iff for every $\epsilon > 0$ there exists a partition P such that $U(P,f,\alpha) - L(P,f,\alpha) < \epsilon$. (6mks)

(c) Find the range of convergence of the series $\sum \frac{x^n}{n!}$. (4mks)