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Transitivity and Imprimitivity of Product of Three Alternating Groups on Cartesian Product of Three Sets of Ordered Quadruples

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Abstract

In this paper, we determine the transitivity and primitivity of the product of three alternating groups acting on the cartesian product of three sets of ordered quadruples. When $n \geq 6$ and using the Orbit-Stabilizer theorem, the action has been determined to be transitive and using the definition of primitivity and blocks, the action has been determined to be imprimitive.

Keywords: *Transitivity action; primitivity action; ordered sets of quadruples; cartesian product and alternating group.*

Mathematics Subject Classification: Pure Mathematics.

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1 Preliminaries

1.1 Introduction

In this paper, the following notations have been used as described: A_n -an alternating group of degree n and order $\frac{n!}{2}$; $|G|$ – the order of a group G; $|G:H|$ -Index of H in G; $T^{[4]}$ – the set of an ordered triple from set $T = \{1,2,3,\ldots,n\}$; $U^{[4]}$ – the set of an ordered triple from set $U = \{n+1, n+2,\ldots,2n\}$; $W^{[4]}$ – the set of an ordered triple from set $W = \{2n+1, 2n+2,\ldots,3n\}$; $[a,b,c,d]$ -Ordered quadruple; $A_n \times A_n \times A_n$ -Cartesian product of alternating group A_n; $T^{[4]} \times U^{[4]} \times W^{[4]}$ -Cartesian product of ordered sets of triples $T^{[4]}$, $U^{[4]}$ and $W^{[3]}$.

The product of three alternating groups, $A_n \times A_n \times A_n$, acts on $T^{[4]} \times U^{[4]} \times W^{[4]}$, by the rule;

$g_1\{([1,2,3,4],\ldots,[n,n-1,n-2,n-3])\} \times g_2\{([n+1,n+2,n+3,n+4],\ldots,[2n,2n-1,2n-2,2n-3])\} \times g_3\{([2n+1,2n+2,2n+3,2n+4],\ldots,[3n,3n-1,3n-2,3n-3])\} = \{([1,2,3,4],\ldots,[n,n-1,n-2,n-3]),([n+1,n+2,n+3,n+4],\ldots,[2n,2n-1,2n-2,2n-3]),([2n+1,2n+2,2n+3,2n+4],\ldots,[3n,3n-1,3n-2,3n-3])\}$.

$\forall\, g_1, g_2, g_3 \in A_n$, $\{([1,2,3,4],[1,2,3,5],\ldots,[n,n-1,n-2,n-3])\} \in T^{[4]}$, set of ordered quadruples from the set $T = \{1,2,3,\ldots,n\}$; $\{([n+1,n+2,n+3,n+4],\ldots,[2n,2n-1,2n-2,2n-3])\} \in U^{[4]}$, set of ordered quadruples from the set

$U = \{n+1, n+2,\ldots,2n\}$; and $\{([2n+1,2n+2,2n+3,2n+4],\ldots,[3n,3n-1,3n-2,3n-3])\} \in W^{[4]}$, set of ordered quadruples from the set $W = \{2n+1, 2n+2,\ldots,3n\}$.

Cameron (1972) worked on "the suborbits of multiply transitive permutations and later in 1973 studied the suborbits of primitive groups".

Ndarinyo *et al.*, (2015) showed that "the alternating group $A_n = 5,6,7$ acts transitively on unordered and ordered triples from the set $P = 1,2,\ldots,n$ when $n \leq 7$ through determination of the number of orbits".

Muriuki *et al.*, (2017) showed that "for the action of direct product of three symmetric groups on Cartesian product of three sets, the action is both transitive and imprimitive for all $n \geq 2$ and the associated rank is 2^3."

Mutua *et al.*, (2026) showed that "the direct product of $S_n \times A_n$, of the symmetric group S_n by the alternating group A_n on the cartesian product $X \times Y$ has its action both transitive and imprimitive when $n \geq 3$." Nyaga (2018) proved that "the direct product action of the alternating group on the Cartesian product of three sets is transitive."

Maraka *et al.*, (2021) showed that "the action of the cartesian product of the alternating group, $A_n \times A_n \times A_n$, on the cartesian product of $P^{[3]} \times S^{[3]} \times V^{[3]}$, the cartesian product of ordered sets of triples is transitive when $n \geq 5$."

Maraka *et al.*, (2022) showed that "the action of the cartesian product of the alternating group, $A_n \times A_n \times A_n$, on the cartesian product of $P^{[3]} \times S^{[3]} \times V^{[3]}$, the cartesian product of ordered sets of triples is imprimitive when $n \geq 5$."

Based on these results we investigate some properties of $A_n \times A_n \times A_n$, acting on $T^{[4]} \times U^{[4]} \times W^{[4]}$.

1.2 Definitions and theorems

Definition 1.2.1: Group action (Kinyanjui *et al.*, 2013): Let T be a non-empty set. A group G is said to act on the left of T if for each $g \in G$ and each $t \in T$ there corresponds a unique element $gt \in G$ such that:

(i) $(g_1 g_2) = g_1(g_2 t)$, $g_1, g_2 \in G$ and $t \in T$.
(ii) For any $t \in T$, $et = T$, where e is the identity in G.

The action of G from the right on T can be defined in the same manner.

Definition 1.2.2: Orbit (Njagi, 2016): Let G act on a set T. Then T is partitioned into disjoint equivalent classes called orbits or transitivity classes of the action. For every $t \in T$ the orbit containing t is called the orbit of t and is denoted by $Orb_G(t)$.

Definition 1.2.3 Fixed point (Kinyanjui *et al.*, 2013): Let G act on a set T. The set of elements of T fixed by $g \in G$ is called the fixed-point set of G and is denoted by $Fix(g)$. Thus $Fix(g) = \{t \in T | gt = t\}$.

Definition 1.2.4: Transitive group (Cameron, 1977): If the action of a group G on set T has only one orbit, then we say that G acts transitively on T. In other words, G acts transitively on T if for every pair of points $t, u \in T$, there exists $g \in G$ such that $gt = u$.

Theorem 1.2.4: (Orbit – Stabilizer Theorem, Rose, 1978, p.72): Let G act on a set T. Then $|Orb_G(t)| = |G: Stab_G(t)|$.

Definition 1.2.5: Blocks and primitivity (Nyaga *et al.,* 2011): "Assume that the action of G on T is transitive. For every subset A of T such that each $g \in G$, let $gA = \{ga: a \in A\} \subseteq T$. A subset A of T is referred to as a block for the action if, for every $g \in G$, either $gA = A$ or $gA \cap A = \emptyset$. $\emptyset, T$ and all singleton subsets of T are definitely blocks known as trivial blocks. If these are the only blocks, then G acts primitively on T otherwise, G acts imprimitively" (Maraka et al., 2022).

Definition 1.2.6: Direct product action (Cameron *et al,* 2008): Let (G_1, P_1) and (G_2, P_2) be permutation groups. The direct product $G_1 \times G_2$ acts on the disjoint union $P_1 \cup P_2$ by the rule $p(g_1, g_2) = \begin{cases} pg_1, if\ p \in P_1, \\ pg_2, if\ p \in P_2 \end{cases}$ and on the Cartesian product $P_1 \times P_2$ by the rule $(p_1, p_2)(g_1, g_2) = (p_1 g_1, p_2 g_2)$ (Higman, 1970).

Theorem 1.2.7: (Armstrong, 1997): The $G_1 \times G_2 \times G_3$-orbit containing $(t, u, w) \in T \times U \times W$ is given by $Orb_{G_1}(t) \times Orb_{G_2}(u) \times Orb_{G_3}(w)$ and the stabilizer of (t, u, w) is given by $Stab_{G_1}(t) \times Stab_{G_2}(u) \times Stab_{G_3}(w)$.

2 Results

Lemma 2.1: The action of the product of three alternating groups $A_6 \times A_6 \times A_6$, on the cartesian product of three sets of ordered quadruples, $T^{[4]} \times U^{[4]} \times W^{[4]}$ is transitive.

Proof: Let $G = A_6 \times A_6 \times A_6$ act on $T^{[4]} \times U^{[4]} \times V^{[4]}$ where;

gap> Arrangements([1,2,3,4,5,6],4);

$T^{[4]}$ = { [[1, 2, 3, 4], [1, 2, 3, 5], [1, 2, 3, 6], ... ,[6, 5, 4, 1], [6, 5, 4, 2], [6, 5, 4, 3]]};

gap> Arrangements ([7,8,9,10,11,12],4);

$U^{[4]}$ = {[[7, 8, 9, 10], [7, 8, 9, 11], [7, 8, 9, 12],. . . . [12, 11, 10, 7], [12, 11, 10, 8], [12, 11, 10, 9]]} and;

gap> Arrangements ([13,14,15,16,17,18],4);

$W^{[4]}$ = {[[13, 14, 15, 16], [13, 14, 15, 17], [13, 14, 15, 18],. . . . [18, 17, 16, 13], [18, 17, 16, 14], [18, 17, 16, 15]]}.

The cartesian product of $T^{[4]} \times U^{[4]} \times W^{[4]}$ is generated using the GAP software with;
$|T^{[4]} \times U^{[4]} \times W^{[4]}| = 46\ 656\ 000$. G is generated by

$< \{(123456), (1234)\}, \{(7\ 8\ 9\ 10\ 11\ 12), (7\ 8\ 9\ 10)\}, \{(13\ 14\ 15\ 16\ 17\ 18), (13\ 14\ 15\ 16)\} >$ using the GAP software. $([1,2,3,4], [7,8,9,10], [13,14,15,16])$ is fixed by an element $(g_t, g_u, g_w) \in G$ if and only if 1,2,3 and 4 comes from a single cycle in g_t; 7,8,9 and 10 comes from a single cycle in g_u and 13,14,15 and 16 comes from a single cycle of g_w.

The $|Stab_G([1,2,3,4], [7,8,9,10], [13,14,15,16])| = 1$.

Using Theorem 1.2.4 (Orbit-Stabilizer Theorem),

$$|Orb_G([1,2,3,4], [7,8,9,10], [13,14,15,16])| = |G: Stab_G([1,2,3,4], [7,8,9,10], [13,14,15,16])|$$

$$= \frac{|G|}{|Stab_G([1,2,3,4], [7,8,9,10], [13,14,15,16])|}$$
$$= \frac{46\ 656\ 000}{1} = 46\ 656\ 000 = |T^{[4]} \times U^{[4]} \times W^{[4]}|$$

Hence, the action of $A_6 \times A_6 \times A_6$ on $T^{[4]} \times U^{[4]} \times W^{[4]}$ is transitive.

Lemma 2.2: The action of the product of three alternating groups $A_7 \times A_7 \times A_7$, on the cartesian product of three sets of ordered quadruples, $T^{[4]} \times U^{[4]} \times W^{[4]}$ is transitive.

Proof: Let $G = A_7 \times A_7 \times A_7$ act on $T^{[4]} \times U^{[4]} \times W^{[4]}$ where;

gap> Arrangements ([1,2,3,4,5,6,7],4);

$T^{[4]}$ = {[[1, 2, 3, 4], [1, 2, 3, 5], [1, 2, 3, 6], [1, 2, 3, 7 ,…. [7, 6, 5, 1], [7, 6, 5, 2], [7, 6, 5, 3], [7, 6, 5, 4]]};
gap> Arrangements ([8,9,10,11,12,13,14],4);

$U^{[4]}$ = {[[8, 9, 10, 11], [8, 9, 10, 12], [8, 9, 10, 13], [8, 9, 10, 14],…,.,.,[14, 13, 12, 8], [14, 13, 12, 9], [14, 13, 12, 10], [14, 13, 12, 11]} and;

gap> Arrangements ([15,16,17,18,19,20,21],4);

$W^{[4]} = \{[[15,16,17,18],\ [15,16,17,19],\ [15,16,17,20],\ [21,20,19,16],\ [21,20,19,17],\ [21,20,19,18]]$

The cartesian product of $T^{[4]} \times U^{[4]} \times W^{[4]}$ is generated using the GAP software with;

$\left| T^{[4]} \times U^{[4]} \times W^{[4]} \right| = 592\ 704\ 000$. G is generated by

$< \{(1234567), (1234)\}, \{(8\ 9\ 10\ 11\ 12\ 13\ 14), (\ 8\ 9\ 10\ 11)\}, \{(15\ 16\ 17\ 18\ 19\ 20\ 21), (15\ 16\ 17\ 18)\} >$ using the GAP software. $([1,2,3,4],[8,9,10,11],[15,16,17,18])$ is fixed by an element $(g_t, g_u, g_w) \in G$ if and only if 1,2,3 and 4 comes from a single cycle in g_t ; 8,9,10 and 11 comes from a single cycle in g_u and 15,16,17 and 18 comes from a single cycle of g_w .

The $|Stab_G([1,2,3,4],[8,9,10,11],[15,16,17,18])| = 27$.

Using the Orbit-Stabilizer Theorem,

$$|Orb_G\,([1,2,3,4],[8,9,10,11],[15,16,17,18])| = |G{:}\,Stab_G([1,2,3,4],[8,9,10,11],[15,16,17,18])|$$

$$= \frac{|G|}{|Stab_G([1,2,3,4],[8,9,10,11],[15,16,17,18])|}$$
$$= \frac{16\ 003\ 008\ 000}{27} = 592\ 704\ 000 = \left|T^{[4]} \times U^{[4]} \times W^{[4]}\right|$$

Hence, the action of $A_7 \times A_7 \times A_7$ on $T^{[4]} \times U^{[4]} \times W^{[4]}$ is transitive.

Theorem 2.3: The action of the product of three alternating groups A_n , $A_n \times A_n \times A_n$, on the cartesian product of three sets of ordered quadruples, $T^{[4]} \times U^{[4]} \times W^{[4]}$ is transitive if $n \geq 6$.

Proof: Let $G = G_p \times G_s \times G_v = A_n \times A_n \times A_n$ act on $T^{[4]} \times U^{[4]} \times W^{[4]}$. It suffices to verify that $\left|T^{[4]} \times U^{[4]} \times W^{[4]}\right|$ is equal to $|Orb_G\,([1,2,3,4],[n+1,n+2,n+3,n+4],[2n+1,2n+2,2n+3,2n+4])|$.

Let $|M| = |Stab_G([1,2,3,4],[n+1,n+2,n+3,n+4],[2n+1,2n+2,2n+3,2n+4])|$.

So, $(g_t, g_u, g_w) \in G = A_n \times A_n \times A_n$ fixes $([1,2,3,4],[n+1,n+2,n+3,n+4],[2n+1,2n+2,2n+3,2n+4]) \in T^{[4]} \times U^{[4]} \times W^{[4]}$ if and only if 1,2,3 and 4 comes from 1-cycle of g_t ; $n+1, n+2, n+3$ and $n+4$ comes from 1-cycle of g_u and $2n+1, 2n+2, 2n+3$ and $2n+4$ comes from 1-cycle of g_w .
The cardinality of $T^{[4]} \times U^{[4]} \times W^{[4]}$ is given as;

$$\left| T^{[4]} \times U^{[4]} \times W^{[4]} \right| = \left(\frac{n!}{(n-4)!}\right)^3$$

The $Stab_G([1,2,3,4],[n+1,n+2,n+3,n+4],[2n+1,2n+2,2n+3,2n+4])$ is isomorphic to: $A_{n-4} \times A_{n-4} \times A_{n-4}$.

Therefore, $|M| = |Stab_G([1,2,3,4],[n+1,n+2,n+3,n+4],[2n+1,2n+2,2n+3,\,2n+4])| = \left|Stab_{G_t}([1,2,3,4]) \times Stab_{G_u}([n+1,n+2,n+3,n+4]) \times Stab_{G_w}([2n+1,2n+2,2n+3,\,2n+4])\right|$

$$|M| = \frac{(n-4)! \times (n-4)! \times (n-4)!}{2 \times 2 \times 2} = \left(\frac{(n-4)!}{2}\right)^3$$

Applying the Orbit-Stabilizer Theorem we get;

$$|Orb_G\,([1,2,3,4],[n+1,n+2,n+3,n+4],[2n+1,2n+2,2n+3,\,2n+4])|$$

$$= |G{:}\,Stab_G([1,2,3,4],[n+1,n+2,n+3,n+4],[2n+1,2n+2,2n+3],2n+4)|$$

$$|G| = \frac{n! \times n! \times n!}{2 \times 2 \times 2} = \left(\frac{n!}{2}\right)^3$$

$$\frac{|G|}{|R|} = \frac{\left(\frac{n!}{2}\right)^3}{\left(\frac{(n-4)!}{2}\right)^3} = \left(\frac{n!}{(n-4)!}\right)^3 .$$

Therefore;

$$\frac{|G|}{|M|} = \left(\frac{n!}{(n-4)!}\right)^3 = \left|T^{[4]} \times U^{[4]} \times W^{[4]}\right|$$

When $n < 6$, the $|Stab_G([1,2,3,4],[n+1,n+2,n+3,n+4],[2n+1,2n+2,2n+3,\,2n+4])| = |A_{n-4} \times A_{n-4} \times A_{n-4}| < 1$.

Since the sets being permuted are finite and A_n is finite, then the order of the stabilizer must finite. The order of the stabilizer is less than 1 for all $n < 6$. Therefore, $n \geq 6$ for the $|Stab_G|$ to be finite.

Hence, $A_n \times A_n \times A_n$ acts transitively on $T^{[4]} \times U^{[4]} \times W^{[4]}$ if $n \geq 6$.

Theorem 2.4: The action of the cartesian product of the alternating group A_n, $A_n \times A_n \times A_n$, on the cartesian product of ordered sets of quadruples, $T^{[4]} \times U^{[4]} \times W^{[4]}$, is imprimitive if $n \geq 6$.

Proof: The action of $A_n \times A_n \times A_n$ on $T^{[4]} \times U^{[4]} \times W^{[4]}$ is transitive by Theorem 2.3 if $n \geq 6$. Consider, $\forall g_1, g_2, g_3 \in A_n$, $\{([1,2,3,4], [1,2,3,5], \ldots, [n, n-1, n-2, n-4], [n, n-1, n-2, n-3])\} \in T^{[4]}$, set of ordered quadruples from the set $T = \{1,2,3, \ldots, n\}$; $\{([n+1, n+2, n+3, n+4], [n+1, n+2, n+3, n+5], \ldots, [2n, 2n-1, 2n-2, ,2n-4], [2n, 2n-1, 2n-2, 2n-3])\} \in U^{[4]}$, set of ordered quadruples from the set $U = \{n+1, n+2, \ldots, 2n\}$; and $\{([2n+1, 2n+2, 2n+3, 2n+4], [2n+1, 2n+2, 2n+3, 2n+5], \ldots, [3n, 3n-1, 3n-2, 3n-4], [3n, 3n-1, 3n-2, 3n-3])\} \in W^{[4]}$, sets of ordered quadruples from the set $W = \{2n+1, 2n+2, \ldots, 3n\}$.

We have;

$L = \{([1,2,3,4], [1,2,3,5], \ldots, [n, n-1, n-2, n-4], [n, n-1, n-2, n-3]) \times ([n+1, n+2, n+3, n+4], [n+1, n+2, n+3, n+5], \ldots, [2n, 2n-1, 2n-2, ,2n-4], [2n, 2n-1, 2n-2, 2n-3]) \times ([2n+1, 2n+2, 2n+3, 2n+4], [2n+1, 2n+2, 2n+3, 2n+5], \ldots, [3n, 3n-1, 3n-2, 3n-4], [3n, 3n-1, 3n-2, 3n-3])\}$.

Let L' be the non-trivial subset of L; $L' = T^{[4]'} \times U^{[4]'} \times W^{[4]'}$ such that $|L'|$ divides $|L|$.

Therefore; $$\frac{\frac{n!}{(n-4)!} \times \frac{n!}{(n-4)!} \times \frac{n!}{(n-4)!}}{\frac{n!}{(n-4)!}} = \frac{|L|}{|L'|}$$

Now, $$L' = \left\{ \begin{array}{c} ([1,2,3,4], [n+1, n+2, n+3, n+4], [2n+1, 2n+2, 2n+4, 2n+5]), \ldots, \\ ([1,2,3,4], [n+1, n+2, n+3, n+4], [3n, 3n-1, 3n-2, 3n-4]), \\ ([1,2,3,4], [n+1, n+2, n+4, n+5], [2n+1, 2n+2, 2n+3, 2n+4]) \end{array} \right\}.$$

So, $|L'| = \frac{n!}{(n-4)!}$

For every element of L' there exist $(g_t, g_u, g_w) \epsilon$ with $\frac{n!}{(n-4)!}$ cycles permutation; $\{([1,2,3,4], [1,2,3,5], \ldots, [n, n-1, n-2, n-4], [n, n-1, n-2, n-3]), ([n+1, n+2, n+3, n+4], [n+1, n+2, n+4, n+5], \ldots, [2n, 2n-1, 2n-2, 2n-4], [2n, 2n-1, 2n-2, 2n-3]), ([2n+1, 2n+2, 2n+3, 2n+4], [2n+1, 2n+2, 2n+4, 2n+5], \ldots, [3n, 3n-1, 3n-2, 3n-4], [3n, 3n-1, 3n-3, 3n-3])\}$ such that for every $(g_p, g_s, g_v) \epsilon G$; $[1,2,3,4]$ is fixed in g_t, $[n+1, n+2, n+3, n+4]$ is fixed in g_u and that $[2n+1, 2n+2, 2n+3, n+4]$ belongs to a single cycle of g_w, then, (g_p, g_s, g_v) either fixes an element of $L' = T^{[4]'} \times U^{[4]'} \times W^{[4]'}$ or takes one element of L' to another so that;

$(g_t, g_u, g_w) T^{[4]'} \times U^{[4]'} \times W^{[4]'} = T^{[4]'} \times U^{[4]'} \times W^{[4]'}$. Any other $(g_t, g_u, g_w) \epsilon G$ moves an element of L' to an element not in L' so that; $(g_t, g_u, g_w)\, T^{[4]'} \times U^{[4]'} \times W^{[4]'} \cap T^{[4]'} \times U^{[4]'} \times W^{[4]'} = \emptyset$. This argument shows that L' is a non-trivial block for the action and the conclusion follows from definition 1.2.5 hence the action is imprimitive if $n \geq 6$.

3 Conclusion

The action of the product of three alternating groups, $A_n \times A_n \times A_n$, $(n \geq 6)$ on the cartesian product of three sets of ordered quadruples, $T^{[4]} \times U^{[4]} \times W^{[4]}$ has been proved to act transitively but imprimitively using the Orbit-Stabilizer theorem and using the definition of primitivity and blocks respectively.

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Competing Interests

Author has declared that no competing interests exist.

References

Armstrong, M. (1997). *Groups and symmetry*. Springer-Verlag.

Cameron, P. J. (1972). Permutation groups with multiply transitive suborbits. *Proceedings of the London Mathematical Society*, *3*(3), 427-440.

Cameron, P. J. (1977). Permutation groups on unordered sets. In *Higher Combinatorics: Proceedings of the NATO Advanced Study Institute held in Berlin (West Germany), September 1–10, 1976* (pp. 217-239). Dordrecht: Springer Netherlands.

Cameron, P. J., Gewurz, D. A., & Merola, F. (2008). Product action. *Discrete mathematics*, *308*(2-3), 386-394.

Higman, D. G. (1970). Characterization of families of rank 3 permutation groups by subdegrees. I. *Archiv der Mathematik, 21*, 151–156.

Kinyanjui, J. N., Musundi, S. W., Rimberia, J., Sitati, N. I., & Makila, P. (2013). Transitivity action of A_n (n = 5, 6, 7, 8) on ordered and unordered pairs. *International Journal of Mathematics and Applications*, 4(9), 77–88.

Maraka, K. M., Musundi, S. W., & Nyaga, L. N. (2021). Transitivity action of the Cartesian product of the alternating group acting on a Cartesian product of ordered sets of triples. *Asian Research Journal of Mathematics, 17*(12), 53–62.

Maraka, K. M., Musundi, S. W., & Nyaga, L. N. (2022). Primitivity action of the Cartesian product of an alternating group acting on a Cartesian product of ordered sets of triples. *London Journals Press, 22*(7), 25–32. https://journalspress.com/primitivity-action-of-the-cartesian-product-of-an-alternating-group-acting-on-a-cartesian-product-of-ordered-sets-of-triples/

Maraka, M. K., Musundi, S. W., & Nyaga, L. N. (2022). Primitivity action of the cartesian product of the alternating group acting on the cartesian product of ordered sets of triples. *London Journal of Research in Science: Natural and Formal, 22*(7), 2631–8504Z.

Muriuki, G. D., Namu, N. L., & Kagwiria, R. J. (2017). Ranks, sub degrees and suborbital graphs of direct product of the symmetric group acting on the cartesian product of three sets. *Pure and Applied Mathematics Journal*, *6*(1), 1.

Mutua, A. K., Nyaga, L. N., & Gachimu, R. K. (2026). On Combinatorial Properties, Invariants and Structures of the Action of An× An on X× Y. *Alupe University Multidisciplinary Research Journal*, *2*(1), 64-69. https://orcid.org/0009-0004-3519-4404

Ndarinyo, C. W., & Rimberia, J. K. (2015). Transitivity action of A_n (n = 5, 6, 7) on unordered and ordered triples. *International Journal of Mathematics and Statistics Education*, 6(7), 21–27.

Njagi, L. (2016). Ranks, subdegrees and suborbital graphs of symmetric group Sn acting on ordered pairs. *Journal of Advance Research in Applied Science (ISSN 2208-2352)*, *3*(2), 32-46.

Nyaga, L. N. (2018). Transitivity of the direct product of the Alternating group acting on the cartesian product of three sets. *International journal of mathematics and its applications*, *6*(1), 889-893.

Rose, J. S. (1978). *A course in group theory*. Cambridge University Press.

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