MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2023/24 ACADEMIC YEAR
FOURTH YEAR *SECOND* SEMESTER

SCHOOL OF SCIENCE
BACHELOR OF SCIENCE (PHYSICS) AND BACHELOR OF EDUCATION SCIENCE

COURSE CODE: PHY 4244-1

COURSE TITLE: STATISTICAL MECHANICS

DATE: **TIME:**

INSTRUCTIONS TO CANDIDATES

1. Answer Question **ONE** and any other **TWO** questions
2. Use of sketch diagrams where necessary and brief illustrations are encouraged.
3. Read the instructions on the answer booklet keenly and adhere to them.

You may require the following useful constants and formulae

$(i)\quad \ln N! = N\ln N - N \qquad \text{and} \qquad N! = \left(\frac{N}{e}\right)^N$

$(ii)\quad \int e^{-ax^2} = \sqrt{\frac{\pi}{a}}$

$(iii)\quad \ln(1\pm x) = 1 \pm x \pm \frac{1}{2}x^2 + \ldots\ldots\ldots\ldots$

$(iv)\quad \int \frac{dx}{e^x+1} = -\ln(1+e^x)$

(v) volume of hypersphere, $V_a = \dfrac{\pi f}{\left(\frac{1}{2}f\right)!}$ where f is degrees of freedom.

(v) Planck's constant, $h = 6.63\times10^{-34}\,J.s$

(vii) mass of an electron, $M_e = 9.11\times10^{-31}\,kg$

$(viii)$ Boltzmann's constant, $K_B = 1.38\times10^{-23}\,JK^{-1}$

(ix) s.h.c of water $= 4.19\text{Jg}^{-1}\text{K}^{-1}$

QUESTION ONE: [20 marks]

a) State the expression that unites thermodynamics and statistical mechanics. Define the terms in the expression. (**2mks**)

b) A classical system is to be examined from a quantum mechanical point of view; State two modifications that may need to be done to achieve this. (**2mks**)

c) Apart from distinguishability/ indistinguishability of identical particles, what are the other differences between classical and quantum statistics. (**4mks**)

d) State four applications of statistical mechanics (**4mks**)

e) Consider the energy of states of an assembly to be $\in_1, \in_2, \ldots, \in_k$ and in the statistical equilibrium, the number of systems assigned to these energy states are $n_1, n_2, \ldots, n_k$ respectively. Obtain expressions for the total number of the systems and total energy of the assembly. (**4mks**)

f) State and explain the basic postulate of statistical mechanics (**2mks**)

g) Define the term "partition function" and state why it is introduced in distribution functions (**2mks**)

QUESTION TWO: [15 marks]

a) In a system of 14 distinguishable particles distributed in two equally probable halves of a box, determine the probability of (7,7) (**4mks**)

b) Under what conditions can Bose-Einsten become Maxwell-Boltzmann distribution? (**2mks**)

c) i. What is chemical potential? (**1mk**)

ii. What is its value in Boltzmann distribution at high temperatures and not too high densities (**1mk**)

iii. what role does it play in the Fermi-Dirac function (**1mk**)

d) Assume the earth's atmosphere is pure nitrogen in thermodynamic equilibrium at a temperature of 300 K. Calculate the height above sea level which the density of the atmosphere is one-half its sea-level value. (**6mks**)

QUESTION THREE: [15 marks]

a) You have two particles to distribute in three states. Use a diagram to show how you achieve this for Fermi-Dirac distribution. What rule guides you in doing this? (**4mks**)

a) The energy *S* is defined as $S = k\log C$ and the most probable distribution is given by $n_i = \omega_i \exp(-\alpha - \beta \in_i)$ where $\beta = \frac{1}{kT}$ and $\exp(\alpha) = \frac{V}{Nh^3}(2\pi mkT)^{3/2}$. Derive an expression for the ideal gas equation given as $PV = NkT$ (**6mks**)

b) Show that the quantity $(2\pi mkT)^{1/2}$ has the character of an average thermal momentum of a particle or system. Knowing this, write down the value of the de Broglie wavelength associated with this system (**5mks**)

QUESTION FOUR: [15 marks]

a) Consider an idealization of a crystal which has *N* lattice points and the same number of interstitial positions (places between the lattice points where atoms can reside). Let *E* be the energy necessary to remove an atom from a lattice site to an interstitial position and let n be the number of atoms occupying interstitial sites in equilibrium.

i. What is the internal energy of the system? **(2mks)**

ii. What is the entropy *S?* Give an asymptotic formula valid when $n \gg 1$ **(5mks)**

iii. In equilibrium at temperature *T,* how many such defects are there in the solid, i.e., what is n? (Assume $n \gg 1$) **(2mks)**

b) Molecules enclosed in a container are in random colliding motion, the Maxwell-Boltzmann distribution gives an idea of finding molecules with speeds between v and $v + dv$. Describe exhaustively the Maxwell-Boltzmann plot. **(6mks)**