



MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS 2023/24 ACADEMIC YEAR FOURTH YEAR SECOND SEMESTER

SCHOOL OF PURE APPLIED AND HEALTH SCIENCES BACHELOR OF SCIENCE

COURSE CODE: MAT 4238-1

COURSE TITLE: TOPOLOGY II

DATE: 18/4/2024

TIME: 1100-1300 HRS

INSTRUCTIONS TO CANDIDATES

- **Question One is Compulsory**
- **Answer Any Other two**

*This paper consists of **THREE** printed pages. Please turn over.*

QUESTION ONE (20 MARKS)

- a) Prove that a topological space X is a T_1 space if and only if every singleton subset of X is closed. **(4 marks)**
- b) Show that every subspace of a Hausdorff space is Hausdorff. **(4 marks)**
- c) State Urysohn's lemma, hence prove that its converse is also true. **(5 marks)**
- d) Prove that every metric space is first countable. **(3 marks)**
- e) Show that the discrete space X is separable if X is countable. **(4 marks)**

QUESTION TWO (15 MARKS)

- a) Show that continuous image of a path-connected set is path connected. **(6 marks)**
- b) Define a separable space, hence show that the set X with cofinite topology is separable any open subspace of a separable. **(5 marks)**
- c) Define homotopy hence show that the following continuous functions where \mathbb{R} is a space of real numbers with standard topology are always homotopic.

$$f : X \rightarrow \mathbb{R}$$

$$g : X \rightarrow \mathbb{R}$$

(4 marks)

QUESTION THREE (15 MARKS)

- a) Show that every T_1 space is a T_0 space. **(4 marks)**
- b) Show that a completely regular space is a regular space **(5 marks)**
- c) Define compactness, hence show that;
- Indiscrete space is compact.
 - The set \mathbb{R} with the usual topology is locally compact but not compact. **(6 marks)**

QUESTION FOUR (15 MARKS)

- a) Prove that any topological space homeomorphic to a connected space is connected. **(6 marks)**
- b) Show that the subspace (Y, τ_Y) of the second countable space (X, τ) is also second countable. **(3 marks)**
- c) Define total disconnectedness hence show that the space X with discrete topology is totally disconnected and locally connected **(6 marks)**

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