

**MAASAI MARA UNIVERSITY  
REGULAR UNIVERSITY EXAMINATIONS  
2023/2024 ACADEMIC YEAR  
FOURTH YEAR SECOND SEMESTER  
SCHOOL OF PURE APPLIED AND HEALTH SCIENCES  
THE DEGREE OF BACHELOR OF SCIENCE IN  
MATHEMATICS AND EDUCATION  
MAT 4237-1: NUMERICAL ANALYSIS III**

**Instructions to candidates:**

*Answer Question 1. And any other TWO.*

*All Symbols have their usual meaning*

DATE:    TIME:

**Question 1(20 Marks)**

- (a) Solve the boundary value problem  $\frac{d^2y}{dx^2} + y + 1 = 0$ ,  $y(0) = y(1) = 0$  (Take  $n=4$ ) (7 Marks)
- (b) Classify the equation  $y^2u_{xx} - 2u_{xy} + u_{yy} - u_y = 8y$  (3 Marks)
- (c) Use Schmidt's explicit formula to solve the partial differential equation  $u_t = u_{xx}$  subject to

$$\begin{aligned} u(x, 0) &= \sin\pi x, \quad 0 \leq x \leq 1 \\ u(0, t) &= u(1, t) = 0 \end{aligned} \quad (1)$$

Carry out computations for two levels taking  $h = \frac{1}{3}$ ,  $k = \frac{1}{36}$  (10 Marks)

**Question 2 (15 Marks)**

Solve  $\Delta u = 0$  under the conditions ( $h=k=1$ )

$$u(0, y) = 0, \quad u(4, y) = 12 + y \text{ for } 0 \leq y \leq 4$$

$$u(x, 0) = 3x, \quad u(x, 4) = x^2, \text{ for } 0 \leq x \leq 4 \quad (15 \text{ Marks})$$

**Question 3 (15 Marks)**

The transverse displacement  $u$  of a point at a distance  $x$  from one end and at any time  $t$  of a vibrating string satisfies the equation  $u_{tt} = 4u_{xx}$ , with boundary conditions

$$u(0, t) = u(4, t) = 0 \quad t > 0 \quad \text{and initial conditions}$$

$$u(x, 0) = x(4 - x)$$

$$u_t(x, 0) = 0, \quad 0 \leq x \leq 4$$

Solve the equation numerically for one half period of vibration, taking

$$h = 1, \quad k = \frac{1}{2} \quad (15 \text{ Marks})$$

**Question 4 (15 Marks)**

Solve the the partial differential equation  $u_{xx} + u_{yy} = -10(x^2 + y^2 + 10)$  over the square with sides  $x = 0 = y$ ,  $x = 3 = y$  with  $u = 0$  on the boundary and mesh length=1. Compute the iterations to the nearest whole number (15 Marks)