

**MAASAI MARA UNIVERSITY  
REGULAR UNIVERSITY EXAMINATIONS  
2023/2024 ACADEMIC YEAR  
SECOND YEAR SECOND SEMESTER  
SCHOOL OF PURE APPLIED AND HEALTH SCIENCES  
THE DEGREE OF BACHELOR OF SCIENCE IN  
MATHEMATICS AND EDUCATION  
MAT 2214-1: ORDINARY DIFFERENTIAL EQUATIONS I**

**Instructions to candidates:**

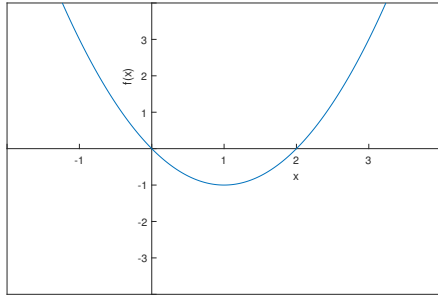
*Answer Question 1. And any other THREE.*

*All Symbols have their usual meaning*

DATE:    TIME:

**Question 1(20 Marks)**

- (a) Given the graph of  $f(x)$  for the differential equation  $\dot{x} = f(x)$



- (i) Draw the phase line and classify the equilibrium points of the differential equation **(2 Marks)**
- (ii) If  $x = 1$ , find  $\lim_{t \rightarrow \infty} x(t)$  **(1 Mark)**
- (iii) If  $x = 3$ , find  $\lim_{t \rightarrow \infty} x(t)$  **(1 Mark)**
- (b) Show that the differential equation  $\dot{x} = \frac{2xt}{t^2 - x^2}$  is homogeneous and determine its degree **(3 Marks)**
- (c) Given the differential equation  $(3t^2x + tx^2)dt + (t^3 + t^2x)dx = 0$ , show that this is an exact differential equation. **(3 Marks)**
- (d) Determine the general solution to the second order differential equation  $x'' - x' - 2 = 0$  **(3 Marks)**
- e Solve the differential equation  $\frac{dt}{y^2x} = \frac{dx}{y^2t} = \frac{dy}{x^2t}$ , where  $t$  is the independent variable by grouping **(4 Marks)**
- (f) Determine the differential equation of the orthogonal trajectories to the family  $y = ce^{-t}$  **(3 Marks)**

**Question 2 (10 Marks)**

Ms. Mukoma a civil servant deposited Ksh. 100,000 in a savings account with interest occurring at a rate of 10% compounded continuously 10 years ago. After this 10 years in December 2024, she will start withdrawing Ksh. 30,000 from this account.

- (i) Write down a differential equation to represent this information **(2 Marks)**

- (ii) Determine the number of years it will take for the money to get depleted (8 Marks)

**Question 3 ( 10 Marks)**

Given a homogeneous equation  $\frac{dx}{dt} = \frac{-2x+5t}{2x+t}$ , solve by introducing a suitable substitution (10 Marks)

**Question 4 (10 Marks)**

Given the autonomous differential equation  $\frac{dx}{dt} = 0.4x(6 - x)(x - 3)$

- (a) Determine the equilibrium points and state their stability (3 Marks)
- (b) Sketch the graph of  $(x, f(x))$  for the differential equation (3 Marks)
- (c) On the  $(t, x)$  plane, draw some solution curves of the differential equation (4 Marks)

**Question 5 ( 10 Marks)**

Determine the particular solution of the second order nonhomogeneous differential equation  $y'' + 4y' + 4y = 4x^2 + 6e^x$  using the method of undetermined coefficients (10 Marks)

**Question 6 ( 10 Marks)**

By first determining the function  $\frac{1}{N} \left( \frac{\partial M}{\partial x} - \frac{\partial N}{\partial t} \right)$  or  $\frac{1}{M} \left( \frac{\partial N}{\partial t} - \frac{\partial M}{\partial x} \right)$  from the differential equation  $(t^2 + x^2 + t)dt + tx dx = 0$ , find the integrating factor and hence solve the resultant exact equation (10 Marks)