MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES

BACHELOR OF SCIENCE EXAMINATION

COURSE CODE: MAT 2212-1

COURSE TITLE: REAL ANALYSIS I

DATE: **TIME:**

INSTRUCTIONS TO CANDIDATES

Answer Question **ONE** and any other **TWO** questions

*This paper consists of **THREE** printed pages. Please turn over.*

QUESTION ONE – 20 MARKS

a) i) Define the term convergent sequence. **(1 Mark)**

ii) Prove that every convergent sequence in $\mathbb{R}$ is bounded. **(4 Marks)**

b) i) Define the term countable set. **(1 Mark)**

ii) Show that the set $\mathbb{Z}$ of all integers is countable. **(2 Marks)**

c) Using the additive axioms of $\mathbb{R}$, $\forall x, y \in \mathbb{R}$, show that:

i) $|x+y| \le |x| + |y|$ **(3 Marks)**

ii) $x(-y) = -(xy)$ **(2 Marks)**

d) Explain the term 'open set' as used in Metric spaces and hence prove that in a metric space (X, ρ) any arbitrary union of open sets is open. **(5 Marks)**

e) Precisely define the term Cauchy sequence. **(2 Marks)**

QUESTION TW0 – 15 MARKS

a) State the following:

i) Monotone Convergence Theorem. **(1 Mark)**

ii) Completeness Axiom for $\mathbb{R}$. **(1 Mark)**

b) Show from the first principles that: $\lim_{n\to\infty}\left(\frac{3n}{2n+1}\right) = \frac{3}{2}$. **(3 Marks)**

c) State and prove Sandwich Theorem for real sequences and hence verify that:

$$\lim_{n\to\infty}\left(\frac{\cos n}{n}\right) = 0$$ **(6 Marks)**

d) Given that $A = \left\{\frac{2n+1}{n+1} : n \in \mathbb{N}\right\}$. Find: $Inf(A)$, $Sup(A)$, $Max(A)$ and $Min(A)$.

(4 Marks)

QUESTION THREE – 15 MARKS

a) i) Prove that a monotonically increasing sequence (x_n) in $\mathbb{R}$ converges if it is bounded above. **(5 Marks)**

ii) From (i) above, deduce that the sequence defined by $(x_n) = \frac{n}{n+1}$ is convergent. **(2 Marks)**

iii) State the limit of convergence of the sequence in (ii) above. **(2 Marks)**

b) State and prove Intermediate Value Theorem and hence use it to deduce that the equation $2x^3 + 5x^2 - 3 = 0$ has at least one real root in the interval $[0,1]$. **(6 Marks)**

QUESTION FOUR – 15 MARKS

a) i) State the absolute value function in $\mathbb{R}$. **(1 Mark)**

ii) Given that $\alpha > 0$, show that $|x| \le \alpha$ iff $-\alpha \le x \le \alpha \ \ \forall x \in \mathbb{R}$. **(4 Marks)**

iii) Use the result in (ii) above to find all values of x for which $\left|x - \frac{1}{2}\right| < \frac{2}{5}$ **(2 Marks)**

b) Show that the function $d(x, y) = |x - y|, \ \ \forall x, y \in \mathbb{R}$ defines a metric on $\mathbb{R}$. **(4 Marks)**

c) Show that if $x, y \in \mathbb{R}$ and $x \ge 0, \ y \ge 0$ then $x < y$ if and only if $x^2 < y^2$. **(4 Marks)**

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