

MAASAI MARA UNIVERSITY REGULAR UNIVERSITY EXAMINATIONS 2023/2024 ACADEMIC YEAR SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS, PHYSICS

MAT:1207-1 ANALYTIC GEOMETRY Instructions to candidates: Answer Question 1 and TWO other Questions . All Symbols have their usual meaning

DATE: TIME:

Question 1(20 Marks)

- (a) With the aid of diagrams where necessary describe the plane Cartesian and the plane polar coordinate systems and show the relationship between them.
 (4 Marks)
- (b) Express the following polar coordinates in Cartesian form and draw a sketch to show the indicated points (4 Marks)
 (i) (5, 30°)
 (ii) (2, 330°)
 (iii) (-2, -120°)
 (iv)(-3, -60°)
- (c) Express the following Cartesian coordinates in polar form (3 Marks) (i) $(1,\sqrt{3})$ (ii) $(-1,-\sqrt{3})$ (iii) $(1,-\sqrt{3})$
- (d) Determine the length of the straight line segment (i) from the (polar) point $(2, \frac{\pi}{6})$ to the point $(4, \frac{7\pi}{6})$ (2 Marks) (ii) from the (Cartesian) point (4, 2) to the point (12, -4) (2 Marks)
- (e) Determine two points of intersection of the curves: $r^2 = 2b^2 \sin 2\theta$, r = bwhere b is a non-zero constant (2 Marks)
- (f) Describe and illustrate(i) right circular cone in 3-space
 - (ii) conic sections

Question 2 (15 Marks)

| (a) | (i) Determine the polar equation of a circle of radius a with centre | |
|-----|--|----------------------|
| | at (b, α) | $(5 \mathbf{Marks})$ |
| | (ii) Draw a sketch with $0 < \alpha < \frac{\pi}{2}$ | (1 Mark) |
| (b) | (i) Determine the equation if the circle in part a passes through | |
| | the origin | $(3 \mathbf{Marks})$ |
| | (ii) Determine the equation if the centre of the circle in part b is | 5 on |
| | the y-axis | $(3 \mathbf{Marks})$ |
| | | |

(c) Write down the equation of a circle of radius 2 passing through the origin with centre on the y-axis (2 Marks)

(3 Marks)

Question 3 (15Marks)

- (a) A conic section has eccentricity e, focus at the origin and a vertical directrix L at a distance d to the left of F. Prove that if the curve is a parabola or an ellipse then the curve lies to the left of the origin and its equation is $r = \frac{ed}{1 e \cos \theta}$ (5 Marks)
- (b) The polar equation of a conic with a focus at the origin and a vertical directrix to the right of F is

$$r = \frac{6}{3 + \cos\theta}$$

Determine (i) the eccentricity e, (ii) the distance from the focus to the directrix Sketch the curve showing its position relative to the origin (6 Marks)

(c) A conic section has eccentricity $\frac{1}{2}$, directrix 3x + 4y = 25 and focus at the origin. Determine the distance from the focus to the directrix and draw a sketch to show the relative location of the curve in the Cartesian xy plane (4 Marks)

Question 4 (15 Marks)

- (a) The equation of a curve is given by $r = r(\theta) = \sqrt{3} \sin \theta$, $0 \le \theta \le 2\pi$
 - (i) Determine values of θ for which $\frac{dr}{d\theta} = 0$
 - (ii) Show that $r(\theta) = r(\pi \theta)$
 - (iii) Determine the minimum value of r
 - (iv) Determine the maximum value of r
 - (v) Write down $r(0), r(\frac{\pi}{2}), r(\pi), r(\frac{3\pi}{2})$

Sketch the curve

(b) Sketch the curve $r = 1 + 2 \sin \theta$ and comment on any similarities and differences with the curve in part (a) (6 Marks)

(9 Marks)