MAASAI MARA UNIVERSITY
REGULAR UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR
FOURTH YEAR SECOND SEMESTER
SCHOOL OF PURE APPLIED AND HEALTH SCIENCES
THE DEGREE OF BACHELOR OF SCIENCE IN APPLIED STATISTICS
STA 4246 MULTIVARIATE ANALYSIS

Instructions to candidates:

Answer Question 1. And any other TWO.

All Symbols have their usual meaning

DATE: TIME:

Question 1(30 Marks)

(a) Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$$

Also check if the normalised eigevectors are orthogonal (**10 Marks**)

(b) (i)

(ii)Given that a random matrix $\mathbf{X}$ is given by

$$\mathbf{X} = \begin{bmatrix} X_{11} & X_{12} & ... & X_{1n} \\ X_{21} & X_{22} & ... & X_{2n} \\ . & & & \\ . & & & \\ . & & & \\ X_{n1} & X_{n2} & ... & Xnn \end{bmatrix}$$

Show that the covariance matrix of $\bar{X}$ is given by

$$Cov(\bar{\mathbf{X}}) = \frac{\mathbf{1}}{\mathbf{n}}\sum \quad \textbf{where}\sum = \begin{bmatrix} \sigma_{11} & \sigma_{12} & ... & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & ... & \sigma_{2n} \\ . & & & \\ . & & & \\ . & & & \\ \sigma_{n1} & \sigma_{n2} & ... & \sigma_{nn} \end{bmatrix}$$

(**4 Marks**)

(c) Let $\mathbf{X_i}$; $i = 1, 2, ..., n$ represent a random vector sample from $N_p\,(\mu, \sigma)$, the joint distribution function of $f(\mathbf{x}$ is given by

$$f(x_1, x_2, ...x_n) = \frac{1}{(2\pi)^{np}|\sum|^{\frac{1}{2}}} exp\left(-\frac{1}{2}\sum_{i=1}^{n}(\mathbf{x_i} - \mu)'\sum{}'(\mathbf{x_i} - \mu)\right)$$

Show that

(i)

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{x_i} = \bar{\mathbf{x}}$$

(ii)

$$\sum_{i=1}^{n} = \sum_{i=1}^{n}(\mathbf{x_i} - \hat{\mu})(\mathbf{x_i} - \mathbf{x})' = \frac{\mathbf{n-1}}{\mathbf{n}}\,\mathbf{S}$$

(**10 Marks**)

Question 2 (20 Marks)
Two different sample nutrients A and B each having sample size $n = 10$ of the same food measured in milligrams per 100 gram portion gave the following data during a laboratory analysis

A	3.17	3.45	3.73	1.82	4.39	2.91	3.54	4.09	2.85	2.05
B	3.45	2.35	5.09	3.88	3.64	4.63	2.88	3.98	3.74	4.36

You are required to test if the sample came from a food with mean nutrient amount. Take $\alpha = 0.05 \quad \mu_{\mathbf{o}} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ (**20 Marks**)

Question 3 (20 Marks) Given the matrix $\mathbf{A} = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$

(a) Find $\mathbf{A}\ \mathbf{A}'$ and $\mathbf{A}'\mathbf{A}$ (**4 Marks**)

(b) Work out the eigenvalues and eigenvectors corresponding to $\mathbf{A}'\mathbf{A}$ (**16 Marks**)

Question 4 (20 Marks)

(a) Given a line through μ of an ellipsoid represented by its coordinates $\mathbf{x}$ on the surface, then the first principal axis will have coordinates that maximized its squared half length $(\mathbf{x} - \mu)'(\mathbf{x} - \mu)$. That is the half length is given by

$$d = \sqrt{(\mathbf{x} - \mu)'(\mathbf{x} - \mu)}$$

Prove that the half length of the major(principal) axis is equal to $d_l = \sqrt{\lambda_l \ c}$ where c is fixed (**10 Marks**)

(b) Given a bivariate normal distribution, $p = 2$ and the covariance matrix

$$\sum = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$$

and assuming that $\sigma_1 1 = \sigma_{22}, \sigma_{12} > 0$. Show that the normalised eigenvectors of $\sum$ are given by $\mathbf{e_1} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$ and $\mathbf{e_2} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$ respectively (**10 Marks**)