MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR
FOURTH YEAR SECOND SEMESTER

SCHOOL OF PURE APPLIED AND HEALTH SCIENCES
BACHELOR OF SCIENCE

COURSE CODE: STA 4243

COURSE TITLE: STOCHASTIC PROCESS

DATE: 18/4/2023 **TIME: 0830-1030 HRS**

INSTRUCTIONS TO CANDIDATES

Answer Question ONE and any other TWO questions

Question One (30 Marks)

a) Define the following terms as used in stochastic processes.

i. State of a process (1 mark)

ii. Discrete time Stochastic process (1 mark)

iii. Poisson process (1 mark)

iv. Markov chain (1 mark)

b) Obtain the generating function of the sequence $\{Y_t = 1\}$, for k. $\{a_k\} = 1,1,1,\ldots$ (3 marks)

c) Consider a random walk process where a particle moves from one state to another according to the following the following transition probabilities;

	E_1	E_2	E_3	E_4	E_5
E_1	1	0	0	0	0
E_2	½	0	½	0	0
E_3	0	¾	0	¼	0
E_4	0	0	¼	½	¼
E_5	0	0	0	0	1

i. Show that the random walk process represents an absorbing Markov Chain. (3 marks)

ii. Determine the fundamental matrix and describe it. (3 marks)

iii. If the process starts in E_2, find the expected number of steps it takes before absorption. (3 marks)

iv. For the situation in iii. Above, find the probability that the process will end in E_5 (2 marks)

d) Let X be a random variable such that $pr(X = k) = p_k$ and $pr(X > k) = q_k = \sum_{r=k+1}^{\infty} p_k$ for $k \geq 0$. Obtain the generating function of $pr(X > k)$ (5 marks)

e) Let $\{Y_t\}_{t\epsilon T}$ be a stochastic process with probability distribution

$$P(Y_t = x) = \begin{cases} pq^{x-1}, & 0,1,2,3,4,5 \ldots \\ 0 & elsewhere \end{cases}$$

Find the probability generating function of the process and hence use the p.g.f to obtain the mean and variance of the process. (7marks)

Question Two (20 Marks)

a) Let $y_1, y_2, \ldots . y_n$ be independent Poison random variables with parameter θ_i,i =1, 2,….Find the distribution of $z = y_1 + y_2 + \cdots y_n$ (5marks)

b) Consider the process $x(t)$ =Acosω t+Bsinω t, where A and B are uncorrelated random variables each with mean 0 and variance 1 and ω is a positive constant. Show that the process $\{x(t)\}$ is covariance stationary. (5marks)

c) Consider the difference differential equation of a birth and death process:

$$p_n' (t) = -\lambda_n p_n(t) + \lambda_{n-1} p_{n-1}(t), \qquad n \geq 1$$

and

$$p_0'(t) = \lambda_0 p_0(t) \qquad n = 0$$

Using the probability generation function technique with initial conditions

$p_n(0)=1 \ for\ n = 1$ and $p_n(0)=0 \ for\ n \neq 1$.

Use the feller's method to find the mean and variance of the process

(10 marks)

Question Three (20 Marks)

a) Explain the following terms as used in Markov Chains.

i. Transient state (1 mark)

ii. Persistent state (1 mark)

iii. An irreducible set of states (1 mark)

b) The terms of the sequence $\{a_n\}$ is generated by:

$$a_n = 5a_{n-1} - 26, \quad n \geq 2 \text{ and } a_0 = 1, \quad a_1 = -2$$

Find the generating function of the sequence, $A(S)$ (2 marks)

c) Write the equation governing the Poisson process at a given time `t' in the birth-and-death process (2 marks)

a) Using the usual notations, show that $\frac{dG}{dt} - \lambda(s-1)G = 0$. (4 marks)

b) Solve the equations given in c) to obtain $G(s,t)$ explicitly, using the initial condition that the population size is of size `a' at time $t = 0$
` (6 marks)

c) Use $G(s,t)$ to work out the mean and variance of the process at time `t'

(3 marks)

Question Four (20 Marks)

a) Consider a birth- death process with difference differential equations:

$$p_n'(t) = -2n\mu p_n(t) + (n-1)\mu p_{n-1}(t) + (n+1)\mu p_{n+1}(t) \qquad n \geq 1$$

and

$$p_0'(t) = \mu p_1(t) \qquad n = 0$$

Suppose the initial conditions:

$p_n(0)=1$ $for\ n = 1$ and $p_n(0)=0$ $for\ n \neq 1$. Obtain the solution of the process.

(13 marks)

b) Let $Z_N = X_1 + X_2 + \ . \ . \ + X_N$ where $X_i's$ are independent and identically distributed as the Poisson random variable given in (a) and N is also a Poisson variable with parameter μ.

i. Show that the probability generating function of Z_N is given by,

$G_n(s) = \exp\{\mu(e^{\lambda(s-1)} - 1)\}$ (5 marks)

ii. Hence or otherwise, deduce the mean value of Z_N. (2 marks)

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