MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2022/2023 ACADEMIC YEAR
THIRD YEAR SECOND SEMESTER

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES
BACHELOR OF SCIENCE APPLIED STATISTICS WITH COMPUTING

COURSE CODE: STA 3229-1
COURSE TITLE: TESTING OF HYPOTHESIS

DATE: **TIME:**

INSTRUCTIONS TO CANDIDATES

1. Answer **Question ONE** and any other **Two** questions.
2. Show all the workings clearly
3. Do not write on the question paper
4. All Examination Rules Apply.

Question One (20 Marks)

(a) Let $X_1, \ldots, X_{10}$ be a random sample of size 10 from the Bernoulli distribution

$$f(x;\theta) = \theta^x (1-\theta)^{1-x}, \quad x = 0,1$$
$$= 0, \qquad elsewhere$$

In testing $H_O : \theta = \frac{1}{2}$ *against* $H_1 : \theta = \frac{1}{4}$, the critical region

$$C = \left\{ (x_1, x_2, ..., x_{10}); \sum_{i=1}^{10} x_i \leq 2 \right\}$$

Is used. Find the significance level α. **(6 marks)**

(b) Let $X_1, \ldots, X_n$ be a random sample of size n from $N(\theta, 1)$. Show that the likelihood ratio principle. For testing $H_o : \theta = \theta_1$, *where* θ_1 *is specified, against* $H_1 : \theta \neq \theta_1$ leads to the inequality $\left| \bar{x} - \theta_1 \right| \geq c.$ **(10 marks)**

(c) A random sample $X_1, \ldots, X_{15}$ from $N(\mu, \sigma^2)$ yielded $s^2 = 0.080$. Letting $\alpha = 0.05$, test $H_o : \sigma^2 = 0.09$ against $H_1 : \sigma^2 \neq 0.09$. **(5 marks)**

(d) $X_1, \ldots, X_{12}$ is a random sample from a bivariate normal distribution which yielded a sample correlation coefficient $r = 0.6$. Test $H_o : \rho = 0$ *against* $H_1 : \rho \neq 0$ *at* $\alpha = 0.05$. **(5 marks)**

Question Two (15marks)

(a) Define the following terms:

(i) Test of statistical hypothesis.

(ii) Critical region.

(iii) Significance level. **(6 marks)**

(b) Let $X_1, \ldots, X_{25}$ be a random sample of size n=25 from $N(\theta, 81)$. To test $H_o : \theta = 90$ against $H_1 : \theta = 86$, the critical region $C = \left\{ (x_1, x_2, ..., x_n); \bar{x} < 87 \right\}$ is used.

Find the

(i) probability of committing type I error

(ii) probability of committing type II error.

(iii) The power of the test **(9 marks)**

Question Three (15 marks)

(a) Let $X_1,\ldots,X_n$ be a random sample from an exponential distribution with mean $\theta.$ Show that the

Likelihood ratio test of $H_o:\theta=\theta_o$ *against* $H_1:\theta\neq\theta_o$ has a critical region of the form

$\sum_{i=1}^{n}x_i\leq c_1$ *or* $\sum_{i=1}^{n}x_i\geq c_2.$ **(8 marks)**

(b) $X_1,\ldots,X_n$ is a random sample from a Poisson distribution with parameter $\theta.$ Derive the likelihood ratio test for $H_o:\theta=\theta_o$ *against* $H_1:\theta<\theta_o.$ **(7 marks)**

Question Four (15 marks)

Given below are values of random variables X and Y

X	8	6	6	8	9	12	2	5	6	8	5	11	10
Y	50	22	14	42	75	93	10	53	51	71	34	28	78

(a) Calculate the least squares regression line $y=\alpha+\beta(x-\bar{x}).$

(b) Test $H_o:\beta=0$ *against* $H_1:\beta\neq 0$ *at* $\alpha=0.05.$ **(15 marks)**