MAASAI MARA UNIVERSITY

SCHOOL OF BUSINESS AND ECONOMICS

REGULAR UNIVERSITY EXAMINATIONS
2023/2024 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

BACHELOR OF SCIENCE IN ECONOMICS AND STATISTICS

COURSE CODE: ECO 3104-1
COURSE TITLE: TEST OF HYPOTHESIS

DATE: DECEMBER 2023 **TIME: 2 HOURS**

INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE (20 MARKS)

a) Distinguish between statistical hypothesis and hypothesis testing as used in testing hypothesis **(2 marks)**

b) Define the most powerful and uniformly most powerful critical regions for testing.

$H_0 : \theta = \theta_0 \quad against \quad H_1 : \theta = \theta_1 \quad where \quad \theta_1 \; and \; \theta_0$ are specified values of the parameter θ. **(2 marks)**

c) Suppose that random variable X is randomly distributed with unknown mean, μ and variance 400. If a random sample of size 25 taken from X, find the power of test in testing $H_0 : \mu = 165 \quad against \quad H_1 : \mu = 162$ given the acceptance region is given by $\varpi = \{x : 161.08 \le \overline{X} \le 168.92\}$ **(3 marks)**

d) Given $X_1, X_2, \ldots, X_n$ be random sample from a population whose density is

$$f(x,0) = \begin{cases} \theta^x (1-\theta)^{1-x} & , x = 0,1 \\ 0 & otherwise \end{cases}$$

Obtain the critical region for testing the hypothesis $H_0 : \theta = \theta_0 \quad against \quad H_1 : \theta > \theta_0 \quad where \quad \theta_0$ is a specified value **(3 marks)**

e) Suppose the two samples of sizes 6 and 7 respectively are randomly selected from two normally distributed populations with variances $\sigma_1^2 \; and \; \sigma_2^2$. Suppose we calculate S_1=4.2, S_2= 2.9. Test the hypothesis that $\sigma_1^2 = \sigma_2^2$ against a two sided alternative at 5% level of significance. **(2 marks)**

f) Let $X_1, X_2, \ldots, X_n$ be the random sample of the size n from X which is distributed as $N(\mu,1)$. To test $H_0 : \mu = \mu_0 \quad against \quad H_1 : \mu = \mu_1$, we reject H_0 whenever $\overline{X} > C$. Find the value of C if n =25 and $\alpha = 5\%$. **(3 marks)**

g) A machine fills milk bottles, the mean amount of milk in each bottle are supposed to be 32 with a standard deviation of 0.06. suppose the mean amount of milk is approximately normally distributed. To check if the machine is operating properly, 36 filled bottles will be chosen at random and the mean amount will be determined.

i. If an $\alpha = 0.05$ test is used to decide whether the machine is working properly, what should the rejection criterion be? **(2 marks)**

ii. Find the power of the test if the true mean takes on the following values: 31.97, 31.99, 32, 32.01, and 32.03. **(2 marks)**

iii. Find the probability of a type II error when the true mean is 32.03. **(1 mark)**

QUESTION TWO (15 MARKS)

a) Let X and Y be two independently distributed random variables with distributions X ~N $[\mu_{1,}\sigma_1^2]$ and Y~ $[\mu_2, \sigma_2^2]$ respectively. Let x_1 x_2 ….. x_m be a random sample of size m from X and y_1, y_2, ….., y_n be Another independent random sample of size n from Y.

Derive the likelihood ratio test for testing H_0: $\mu_2 = \mu_1$ against H_1: $\mu_1 \neq \mu_2$.

Assuming $\sigma_1^2 = \sigma_2^2 = \sigma^2$ **(7 marks)**

b) The following statistics were obtained from data drawn from two independent populations X and Y which are normally distributed as follows: X ~N [μ_1, σ_m^2]
and Y~ [μ_2, σ_n^2]

$$\overline{X} = 1.02, \qquad \sum_{i=1}^{m}(X_1 - \overline{X})^2 = 2.44,\ m = 11$$

$$\overline{Y} = 1.66, \qquad \sum_{i=1}^{n}(Y_1 - \overline{Y})^2 = 4.23,\ n = 13$$

Test H_0: $\mu_1 = \mu_2$ against H_1: $\mu_1 \neq \mu_2$. Use $\propto =$ 5% **(3 marks)**

c) A study was conducted of the effects of a special class designed to aid students with verbal skills. Each child was given a verbal skills test twice, both before and after completing a 4-week period in the class. Let Y = score on exam at time 2 - score on exam at time 1. Hence, if the population mean μ for Y is equal to 0, the class has no effect, on the average. For the four children in the study, the observed values of Y are 8-5=3, 10-3=7, 5-2=3, and 7-4=3 (e.g. for the first child, the scores were 5 on exam 1 and 8 on exam 2, so Y = 8-5=3). It is planned to test the null hypothesis of no effect against the alternative hypothesis that the effect is positive, based on the following results from a computer software package:

Variable	No. of cases	Mean	Std.dev	Std.Error
Y	4	4	2.000	1.000

i. Set up the null and alternative hypotheses **(1 marks)**

ii. Calculate the test statistic, and indicate whether the P-value was below 0.05, based on using the appropriate table **(2 marks)**

iii. Make a decision, using P_value of = .05. Interpret. **(2 marks)**

QUESTION THREE (15 MARKS)

a) Consider a simple regression model

$$y_i = \alpha + \beta x_i + e_i \ ,$$

Where α, β are constants and $e_i \sim N(0, \sigma^2)$

Derive a test statistic for testing the hypothesis $H_0 : \beta = 0 \quad against \quad H_1 : \beta \neq 0$. Take the significance level to be $\alpha = 5\%$ **(8 marks)**

b) Test the hypothesis $H_0 : \beta = 0 \quad against \quad H_1 : \beta \neq 0$ at $\alpha = 5\%$ if observations are (1, 2), (3, 2), (2,10), (8, 6) **(2 marks)**

c) Four different brands of margarine were analyzed to determine the level of some unsaturated fatty acids. The data are shown below.

Brand	Fatty Acids (%)				
A	13.5	13.4	14.1	14.2	
B	13.2	12.7	12.6	13.9	
C	16.8	17.2	16.4	17.3	18.0
D	18.1	17.2	18.7	18.4	

i) Perform an appropriate non parametric test at $\alpha = 0.05$. **(5 marks)**

QUESTION FOUR (15 MARKS)

a) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a normal population with unknown mean μ and unknown variance σ^2. Obtain the likelihood ratio critical region for testing the hypothesis

$H_0 : \sigma^2 = \sigma_O^2 \quad against \quad H_1 : \sigma^2 \neq \sigma_0^2$, where σ_0^2 is specified. **(7 marks)**

b) A random sample of n = 7 observations from a normal population produced the following measurements 4,0,6,3,3,5,7. Do the data provide sufficient evidence to indicate that $\sigma^2 < 1$? Take α to be 5%. **(3 marks)**

c) A forensic pathologist wants to know whether there is a difference between the rate of cooling of freshly killed bodies and those which reheated, to determine whether you can detect an attempt to mislead a Coroner about the time of death. He tested several mice for their cooling constant both when the mouse was originally killed and when after the mouse was reheated. The results are presented in the table below.

Freshly Killed	400	395	450	402	345	490	450	432	367	487		
Reheated	378	321	387	400	389	402	354	389	355	376	410	360

The distribution of the differences is unknown. Is there any difference in the cooling constants between freshly killed and reheated corpses? **(5 marks)**