MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR
YEAR FOUR FIRST SEMESTER

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES
BACHELOR OF SCIENCE IN APPLIED STATISTICS & COMPUTING

COURSE CODE: STA 4138
COURSE TITLE: STATISTICAL COMPUTING

DATE: **TIME:**

INSTRUCTIONS TO CANDIDATES

1. Answer Question **ONE and any other Two** questions.
2. Show all the workings clearly
3. Do not write on the question paper
4. All Examination Rules Apply.

Question One (30 Marks)

(a) A random variable X has probility density function given by $f(x) = 3x^2$, $0 < x < 1$.

(i) Write down the Cumulative Distribution Function of X **(3 Marks)**

(ii) Write down an algorithm to simulate sample of size n=20 from the distribution of X using the inverse transformation method.

(3 Marks)

(b) Write down an R function that performs a sample t-test for testing $H_0: \mu_1 - \mu_2 = 0$ vs $H_A: \mu_1 - \mu_2 \neq 0$ at $\alpha = .05$ **(4 Marks)**

(c) (i) Evaluate $I = \int_0^{\pi/2} \sin 2x \, \mathrm{dx}$. **(3 Marks)**

(ii) Write down an R-code for approximating the integral $I = \int_0^{\pi/2} \sin 2x \, dx$ using simulations from the uniform distribution.

(3 Marks)

(d) (i) Write down an R code for simulating the rolling of a fair die.

(3 Marks)

(ii) Outline briefly how you would test that indeed this process mimics the rolling of a die. **(3 Marks)**

(e) Write an R program that will find the zeros of $f(x) = x^3 - 2x - 2$ using the Newton Raphson method. **(8 Marks)**

Question Two (20 Marks)

(a) Nine students were randomly selected who had taken the examination twice. A researcher would like to test the claim that students who take the examination a second time score higher than their first test.

Student	A	B	C	D	E	F	G	H	I	
First EXAM Score	48	51	53	54	55	56	60	62	66	
Second EXAM Score	46	50	53	52	58	58	56	64	69	

(i) What are the hypotheses? **(1 Mark)**

(ii) What are the test's assumptions? **(2 Marks)**

(iii) Write the R command to test of whether the assumption of normality is reasonable.

(iv) Write the R command to perform the hypothesis test. **(2 Marks)**

(v) If the p-value = 0.8981, State your final conclusion in words. Use ⍰ = 0.05. **(2 Marks)**

(vi) Assume that the above data were from two independent populations. State the hypotheses and Write the R commands to test the hypotheses. **(3 Marks)**

(b) The following table lists the fuel consumption (miles/gallon) and weight (lbs) of a vehicle.

Weight (lbs)	3175	3450	3225	3985	2440	2500	2290
MPG(Miles/Gallon)	27	29	27	24	37	34	37

A statistical test was carried out to investigate the linear relationship between Weight and MPG .The p-value was 0.001387 and correlation coefficient was -0.9439269.

(i) Write R command which was used to perform the above test. **(2 Marks)**

(ii) Is the linear relationship statistically significant? Use α=0.05. **(3 Marks)**

(iii) What percent of a vehicle's fuel consumption can be explained by its weight? **(2 Marks)**

(iv) Write an R command that would be used to obtain the linear equation. **(2 Marks)**

Question Three (20 Marks)

The binomial probability mass function with parameters *(n, p)*, 0 < *p* < 1, is given by

$$p_j = P(X = j) = \frac{n!}{(n-j)!j!} p^j (1-p)^{n-j}; \text{ for } j = 0,1,2,\dots,n$$

(a) Verify the recursive relation $p_{j+1} = \frac{(n-j)}{(j+1)} \times \frac{p}{(1-p)} p_j$ **(8 Marks)**

(b) Use the relation in part (a) to write an algorithm for generating binomial random variables. **(6 Marks)**

(c) Write an R code to execute the algorithm in part (b) **(6 Marks)**

Question Four (20 Marks)

(a) The Weibull distribution function with parameters shape=λ scale =1 is of the form

$F(x) = 1 - \exp(x^{\lambda})$

(i) Write an R code to Generate n random numbers from this Weibull distribution **(5 Marks)**

(ii) Write an algorithm that will maximize the log likelihood using the Newton-Raphson algorithm. **(5 Marks)**

(b) Using the Newton Raphson Method in determining the root of the equation $2x - 3\sin(x) - 5 = 0$,

(i) Show that the $(n+1)^{th}$ better approximation to the root is given by

$$x_{n+1} = \frac{3\sin(x_n) - 3x_n\cos(x_n) + 5}{2 - 3\cos(x_n)}$$

(5 Marks)

(ii) Use the results in b(i) to write an R code that can be used to obtain the root **(5 Marks)**