MAASAI MARA UNIVERSITY

MAIN EXAMINATION 2018/2019 ACADEMIC YEAR
SECOND YEAR SECOND SEMESTER EXAMINATIONS

FOR

THE DEGREE OF BACHELOR OF SCIENCE

MAT 2213: CLASSICAL MECHANICS

DATE : **TIME:**

DURATION: 2HRS

INSTRUCTIONS TO CANDIDATES

1. **This paper contains** FOUR **(4) questions**
2. **Answer question** ONE (1) **and any other** TWO **(2) questions**
3. **Do not forget to write your Registration Number.**

QUESTION ONE (30 MARKS)

a) Show that the acceleration **a** of a particle which travels along a space curve with velocity **v** is given by

$$\mathbf{a} = \frac{dv}{dt}\mathbf{T} + \frac{v^2}{R}\mathbf{N}$$ where **T** is the unit tangent vector to the space curve, **N** is the unit principal normal and R is the radius of curvature. **(4mks)**

b) Find the impulse developed by a force given by $\mathbf{F} = 4t\mathbf{i} + (6t^2 - 2)\mathbf{j} + 12\mathbf{k}$ from t=0 to t=2. **(3mks)**

c) A particle moves from rest in a circular path of a circle of radius 20 cm. If its tangential speed is 40 cm/sec, calculate its angular velocity, angular acceleration and normal acceleration. (**5mks)**

d) The angular momentum of a particle is given as a function of time t by

$\Omega = 6t^2\mathbf{i} - (2t+1)\mathbf{j} + (12t^3 - 8t^2)\mathbf{k}$. Find the torque at time t=1. **(3mks)**

e) An object of mass m is dropped from a height H above the ground. Prove that if air resistance is negligible, then it will reach the ground in

i. in a time $\sqrt{2H/g}$ **(2mks)**

ii. with speed $\sqrt{2gH}$ **(2mks)**

f) Given a space curve C with position vector $\mathbf{r} = 3\cos 2t\mathbf{i} + 3\sin 2t\mathbf{j} + (8t - 4)\mathbf{k}$. Find the

i. Curvature **(4mks)**

ii. Unit principal normal to any point of the space curve **(3mks)**

g) A particle of mass m moves along the x axis under the influence of a conservative force field having potential V(x). If the particle is located at positions x_1 and x_2 at respective times t_1 and t_2, prove that if E is the total energy,

$$t_2 - t_1 = \sqrt{\frac{m}{2}}\int_{x_1}^{x_2} \frac{dx}{\sqrt{E - V(x)}}$$ **(4mks)**

QUESTION TWO (20 MARKS)

a) At time t=0 a parachutist having weight of magnitude mg is located at z=0 and is travelling vertically downward with speed v_0. If the force or air resistance acting on the parachute is proportional to the instantaneous speed, find the

i. Speed **(5mks)**

ii. Distance travelled **(3mks)**

iii. Acceleration at any time t>0 **(2mks)**

iv. Show that the parachutist approaches a limiting speed given by $\frac{mg}{\beta}$., where β is an arbitrary constant . **(2mks)**

b) Find the work done in moving a particle once around a circle C in the xy plane, if the circle has center at the origin and radius 3 and if the force field is given by
$\mathbf{F} = (2x - y + z)\mathbf{i} + (x + y - z^2)\mathbf{j} + (3x - 2y + 4z)\mathbf{k}$ **(8mks)**

QUESTION THREE (20 MARKS)

a) Determine the motion of a simple pendulum of length L and mass m assuming small vibrations and no resisting forces. Hence determine the period and amplitude and frequency of vibrations. **(13mks)**

b) A cannon has its maximum range given by $R_{\max}$. Prove that

i. The height reached in such a case is $\frac{1}{4}R_{\max}$ **(4mks)**

ii. the time of flight is $\sqrt{2R_{\max}/g}$ **(3mks)**

QUESTION FOUR (20 MARKS)

a) i) Show that $\mathbf{F} = \left(2xy + z^3\right)\mathbf{i} + x^2\mathbf{j} + 3xz^2\mathbf{k}$ is a conservative force field. **(3mks)**

ii) Find the scalar potential. **(4mks)**

iii) Find the work done in moving an object in this field from (1,-2,1) to (3,1,4). **(3mks)**

b) A spring of negligible mass, suspended vertically from one end, is stretched distance of 20cm when a 5g mass is attached to the other end. The spring and mass are placed on a horizontal frictionless table with the suspension point fixed. The mass is pulled away a distance 20cm beyond the equilibrium position O and released. Find

i. The differential equation and initial conditions describing. the motion. **(3mks)**

ii. The position at any time t, **(4mks)**

iii. The amplitude, period and frequency of the vibrations. **(3mks)**