MAASAI MARA UNIVERSITY

MAIN EXAMINATION 2018/2019 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER EXAMINATIONS

FOR

THE DEGREE OF BACHELOR SCIENCE IN MATHEMATICS

MAT 416: FUNCTIONAL ANALYSIS I

DATE: 26TH APRIL 2019 TIME: 0830 – 1030 HRS

INSTRUCTIONS TO CANDIDATES

1. This paper contains **FOUR** (4) questions
2. Answer question **ONE (1)** and any other **TWO** (**2**) questions
3. Do not forget to write your Registration Number.

QUESTION ONE (30MARKS)

a) Define the following terms

i)	A Banach space	**1mark**
ii)	Strongly convergence of a sequence	**1mark**
iii)	A Hilbert space	**1 mark**
iv)	Radius of convergence of a series	**1 mark**

b) Show that an integral operator is a bounded linear transformation. **5marks**

c) Show that $\left[\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right),\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\right),\left(\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}},-\frac{2}{\sqrt{6}}\right)\right]$ is an orthonormal set

5marks

d) Define a normed linear space and show that if X is an inner product space, then $\|x\| = \langle x, x\rangle^{\frac{1}{2}}$ defines a norm on X **5marks**

e) In the polynomial space p^2 the inner product is given as $\langle u,h\rangle = \int_0^1 u(t)h(t)dt$.

if $u(t) = t+2$ and $h(t) = t^2 - 2t + 3$. Find

i. $\langle u,h\rangle$ ii. $\|u\|$ iii. $\|h\|$ **8marks**

e) Given that $x = \sum_{\alpha\in\Lambda} \left|\langle x, z_\alpha\rangle z_\alpha\right| \quad \forall\ x \in H$. Show that $\|x\|^2 = \sum_{\alpha\in\Lambda} \left|\langle x, z_\alpha\rangle\right|^2$ **3marks**

QUESTION TWO (20MARKS)

a) Show that the differential operator $T: C_{[a,b]} \to C_{[a,b]}$ defined by $Tx(t) = x'(t)$ is an unbounded linear transformation **5marks**

b) State and prove the Reisz representation theorem **10marks**

c) Let $a \in \mathbb{R}^3$. Define $f : \mathbb{R}^3 \to \mathbb{R}$ by $f(x) = \langle x, a\rangle$ for all $x \in \mathbb{R}^3$. Show that f is a bounded linear functional with $\|f\| = \|a\|$ **5marks**

QUESTION THREE (20MARKS)

a) Define bounded linear transformation. **3marks**

b) Show that $\|x+y\|^2+\|x-y\|^2=2\|x\|^2+2\|y\|^2$ **3marks**

c) If $(\langle . \rangle, X)$ is an inner product space, show that for all $x, y \in X$ we have $|\langle x,y\rangle|^2 \le \langle x,y\rangle\langle x,y\rangle$ **7marks**

d) Show that E is closed with respect to the Hilbert space H if and only if it is a complete orthonormal subset **7marks**

QUESTION FOUR (20MARKS)

a) Find the radius of convergence of the series

$$\sum_{n=1}^{\infty}\frac{n\,x^n}{2^{n+1}}$$ **5marks**

b) State and prove the projection theorem **6marks**

c) Show that u for all $f,g \in L_2(a,b)$ the $\langle f,g\rangle=\int_a^b f(x)\,\overline{g}(x)dx$ defines an inner product on $L_2(a,b)$. **4marks**

d) Suppose X and Y are Banach spaces and that T is a bounded linear operator from X to Y **.** If T maps X on to Y**,** show that $T(G)$ is open in Y whenever G is open in X. **5marks**

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