MAASAI MARA UNIVERSITY

REGULAR UNIVERSITY EXAMINATIONS
2018/2019 ACADEMIC YEAR
YEAR II SEMESTER II

SCHOOL OF MATHEMATICAL AND PHYSICAL SCIENCES
BACHELOR OF SCIENCE

COURSE CODE:STA 2217
COURSE TITLE: MATHEMATICAL STATISTICS II

DATE: **TIME:**

INSTRUCTIONS TO CANDIDATES

1. Answer Question **ONE** and any other TWO questions.
2. All Examination Rules Apply.

QUESTION ONE (30 MARKS)

a) (i) Define the term order statistics. (2mks)

(ii) Let X_1, X_2 be a random sample from a distribution with density function.

$f(x) = e^{-x}, 0 < x < \infty$

What is the density of Y= min { X_1, X_2 }. (3mks)

(iii) Consider 2 independent and identically distributed random variables X and Y whose pdfs are;

$f(x) = 6x(1-x), 0 < x < 1$ and

$f(y) = 3y^2, 0 < y < 1$ respectively.

Find the pdf of $Z = XY$. (5mks)

b) The bivariate probability distribution of the random variables X and Y is summarized in the following table.

X \ Y	0	1	2	3
0	k	6k	9k	4k
1	8k	18k	12k	2k
2	k	6k	9k	4k

(i) Find k. (3mks)

(ii) Obtain the marginal distributions of X and Y. (4mks)

(iii) Find the conditional distribution of X given Y =2. (3mks)

(iv) State with a reason whether or not X and Y are independent. (2mks)

c) The daily number of road traffic accidents, Y, in a certain town can be modelled by a Poisson distribution which has probability mass function.

$$P(Y = k) = \frac{e^{-\lambda}\lambda^k}{k!}, k = 0,1,2,...; \lambda > 0$$

(i) Show that the probability generating function (pgf) of Y is $e^{-\lambda(1-t)}$. (3mks)

(ii) Use the pgf to show that E(Y) = Var (Y) = λ. (5mks)

QUESTION TWO (20 MARKS)

(a) The joint probability density function of the random variables X and Y.

$$f(x,y)=\frac{1}{2\pi}\exp\{-\frac{1}{4}(x-1)^2-(y-\frac{1}{4}(1+x))^2\},-\infty<x,y<\infty$$

(i) Use integration to show that X has the normal distribution with mean 1 and variance 2.

(7mks)

(ii) Use integration to show that the moment generating function of X is $M_X(t)=\exp\{t+t^2\}$

(7mks)

(iii) Use the moment generating function to find $E(X^3)$. (6mks)

QUESTION THREE (20 MARKS)

a) Define the terms probability generating function (pgf) and the moment generating function (mgf) of a random variable X and give the relationship between these two functions.

(3mks)

b) The random variable X has the binomial distribution with parameters $n(n>3)$ and $p(0<p<1)$.

(i) Show that the probability generating function of X is; (4mks)

$$\pi t=(pt+1-p)^n,-\infty<t<\infty$$

(ii) Use (i) to show that $E(X)=np$ and $Var(X)=np(1-p)$. (5mks)

(iii) Find $E(X^2)$. (3mks)

(iv) Now suppose that $X_1,X_2,...,X_m$ are independent random variables and X_i has the binomial distribution with parameters n and p for $i=1,2,...,m$. Let $Y=\sum_{i=1}^{m}X_i$. Find the pgf of Y, and hence deduce the distribution of Y. (5mks)

QUESTION FOUR (20 MARKS)

a) Suppose that $X_1 \sim B(n_1, p)$ and $X_2 \sim B(n_2, p)$ independently. Find the probability function of $Y = X_1 + X_2$ (5 marks)

b) Consider a random vector with mean $\mu = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}$ and $\sum = \begin{pmatrix} 3 & -\frac{3}{2} & 0 \\ -\frac{3}{2} & 1 & \frac{1}{2} \\ 0 & \frac{1}{2} & 1 \end{pmatrix}$.

Find the mean vector and variance of the linear combination

$$Z_1 = 2X_1 + 2X_2 - X_3$$
$$Z_2 = X_1 - X_2 + 3X_3$$

(5 marks)

c) Suppose that $\sum$ is a 4×4 covariance matrix of a random vector $\underline{X} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$. Partition $\underline{X}$ such that

(i) $\underline{X}_1 = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ and $\underline{X}_2 = \begin{pmatrix} X_3 \\ X_4 \end{pmatrix}$ (5 marks)

(ii) $\underline{X}_1 = \begin{pmatrix} X_2 \\ X_3 \end{pmatrix}$ and $\underline{X}_2 = \begin{pmatrix} X_4 \\ X_1 \end{pmatrix}$ (5 marks)

QUESTION FIVE (20 MARKS)

a) Derive the probability density function of a random variable X that follows a t-distribution. (10 marks)

b) Derive the probability density function of a random variable X that follows an F-distribution. (10 marks)